The Story of Sandra and Linear Programming

By Tracy Guo, NBHIS

Sandra is a ninth-grade student, who just finished her unit on linear programming. She learned that linear programming is an important method in optimization. Meanwhile, this method is commonly used in life, such as looking for how to sell things to get the maximum profit.

Sandra recalled the 4 steps to solve linear programming problems:

1. Identify the variables: x and y.

2. Identify the constraints. Write the corresponding inequalities according to the conditions.

3. Graph the system of inequalities. The feasible region of the graph containing the solution to the system of linear inequalities.

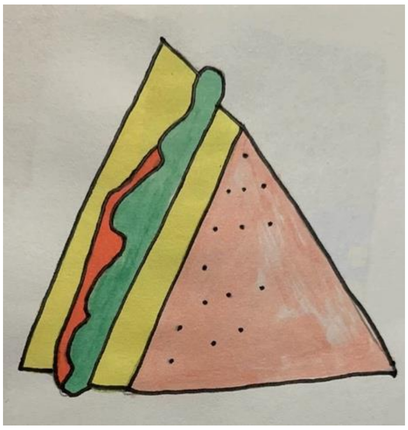
4. Write an objective function to test the corner points of the feasible region to find the optimal solution. The objective function is a linear function, which represents the cost, profit or other constrained quantity to be optimized (maximized or minimized).

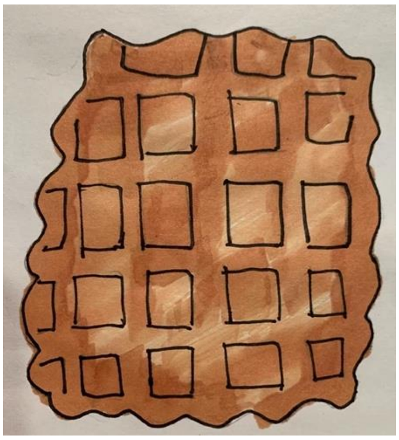
Since Sandra was so interested in this unit, she planned to put linear programming into practice in life to enhance her mastery of it. She thought of her grandmother who worked in a snack bar. In order to make more profits for grandmother, Sandra decided to use linear programming to help grandmother find the best way to sell.

One day, Sandra went to her grandmother's snack bar and told her that she could use mathematical knowledge to make some good suggestions for sales so as to make the most profit. Sandra's grandmother was very happy.

"Can you tell me something about what you sell?" Sandra asked.

"Of course," Sandra's grandmother said. "I make and sell sandwiches and waffles every day. To keep my business going, I have to sell at least 70 sandwiches but cannot make more than 170. I also have to sell at least 50 waffles but cannot make more than 120. In addition, my snack bar cannot make more than 250 items in total. The profit of a sandwich is 0.6 pounds and that of a waffle is 0.4 pounds. How many of each item should I sell to make the maximum profit?"





Sandra immediately started with the 4 steps she had learned:

1. Identify the variables: x and y.

x: number of sandwiches sold

y: number of waffles sold

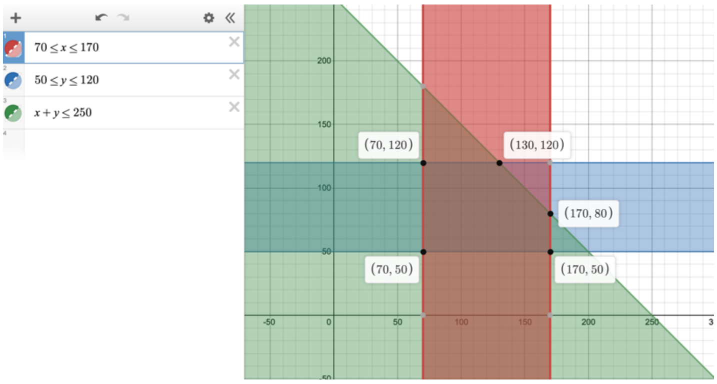
2. Identify the constraints.

At least 70 sandwiches need to be sold and no more than 170 can be made. This is expressed mathematically as $70 \leqslant x \leqslant 170$.

At least 50 waffles need to be sold and no more than 120 can be made. This is expressed mathematically as $50 \leqslant x \leqslant 120$.

No more than 250 items can be made in total. This is expressed mathematically as $x+y \leqslant 250$.

3. Graph the system of inequalities.



4. Write an objective function to test the corner points of the feasible region to find the optimal solution.

Profit will be the objective function: $Z=0.6x+0.4y$. (Find the maximum value)

Bring the 5 corner points into the objective function:

Z=0.6x70+0.4x50=62 Z=0.6x70+0.4x120=90 Z=0.6x130+0.4x120=126

Z=0.6x170+0.4x80=134 Z=0.6x170+0.4x50=122

"I got it!" Sandra told her grandmother. "You should sell 170 sandwiches and 80 waffles to make the maximum profit!"

Sandra passed a pet shop on her way home from the snack bar. She saw a guinea pig in it and decided to buy it.

"Let me tell you how to feed your guinea pig," the shop keeper said. "To ensure ideal health, you need to feed your guinea pig at least 18g of fat, 30g of carbohydrates, and 5g of protein a day. But your guinea pig should not be fed more than 6 ounces a day. Moreover, we have two kinds of feed specially for guinea pigs in our shop, namely feed X and feed Y. So that you can buy and blend them for the best combination. Feed X contains 6g of fat, 10g of carbohydrates, and 1g of protein per ounce, and the price per ounce is 0.3 pounds. Feed Y contains 9g of fat, 10g of carbohydrates, and 3g of protein per ounce, and the price per ounce is 0.5 pounds."

"I see. Now I need to find the best mix of feed X and feed Y," Sandra said.

1. Identify the variables: x and y.

x: number of ounces of feed X purchased

y: number of ounces of feed Y purchased

2. Identify the constraints.

Feed X has 6g of fat, feed Y has 9g of fat, at least 18g need to be fed. This is expressed mathematically as $6x+9y\geqslant 18$.

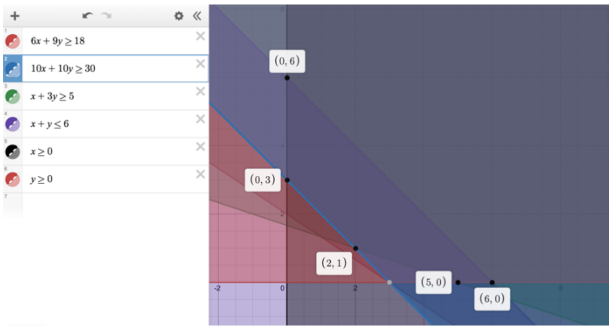
Feed X has 10g of carbohydrates, feed Y has 10g of carbohydrates, at least 36g need to be fed. This is expressed mathematically as $10x+10y\geqslant 30$.

Feed X has 1g of protein, feed Y has 3g of protein, at least 5g need to be fed. This is expressed mathematically as $x+3y\geqslant 5$.

Do not feed more than 6 ounces. This is expressed mathematically as $x+y\leqslant 6$.

Also, both x and y are greater than or equal to 0, so $x\geqslant 0$ and $y\geqslant 0$.

3. Graph the system of inequalities.



4. Write an objective function to test the corner points of the feasible region to find the optimal solution.

Cost will be the objective function: $Z=0.3x+0.5y$. (Find the minimum value)

Bring the 5 corner points into the objective function:

Z=0.3x0+0.5x3=1.5 Z=0.3x0+0.5x6=3 Z=0.3x6+0.5x0=1.8

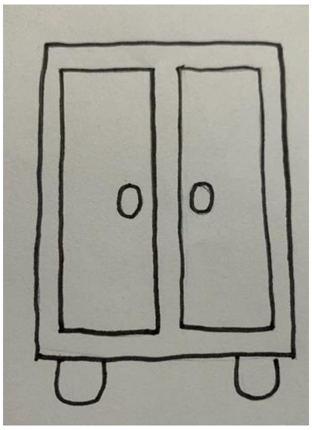
Z=0.3x5+0.5x0=1.5 Z=0.3x2+0.5x1=1.1

"Well, I need to buy the optimal blend of 2 ounces of feed X and 1 ounce of feed Y for my guinea pig," Sandra said.

When Sandra got home, she told her mother that she used linear programming to find the optimal selling plan for grandmother and the best food mixing for her guinea pig.

"Awesome!" Sandra's mother said. "I was wondering if you could help me with what I am doing right now. I am buying cupboards online since our cupboards are old. There are cupboards in two different colors. A white cupboard costs 10 pounds. It needs 3 square feet of floor space and has 6 square feet of storage space. A brown cupboard costs 40 pounds. It needs 6 square feet of floor space and has 20 square feet of storage space. I have 120 pounds to buy these cupboards. Furthermore, the floor area of our house where we can put cupboards is no more than 30 square feet. How many of each item should I buy to maximize storage volume?"

"I will work it out for you," Sandra said.



1. Identify the variables: x and y.

x: number of white cupboards purchased

y: number of brown cupboards purchased

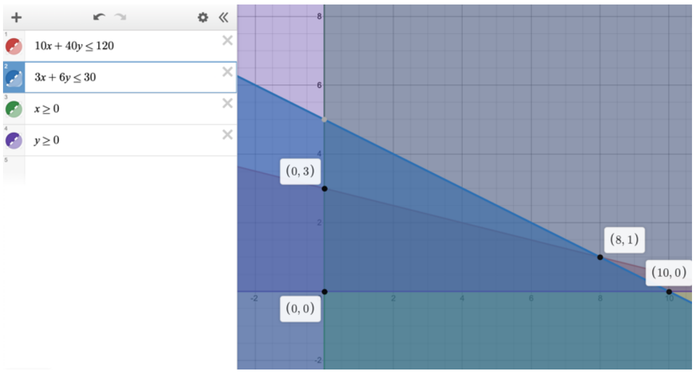
2. Identify the constraints.

A white cupboard costs 10 pounds, a brown cupboard costs 40 pounds, cannot spend more than 120. This is expressed mathematically as $10x+40y\leqslant 120$.

A white cupboard needs 3 square feet, a brown cupboard needs 6 square feet, the available floor area is no more than 30. This is expressed mathematically as $3x+6y\leqslant 30$.

Also, both x and y are greater than or equal to 0, so $x\geqslant 0$ and $y\geqslant 0$.

3. Graph the system of inequalities.



4. Write an objective function to test the corner points of the feasible region to find the optimal solution.

Volume will be the objective function: $Z=6x+20y$. (Find the maximum value)

Bring the 4 corner points into the objective function:

Z=6x0+20x0=0 Z=6x0+20x3=60

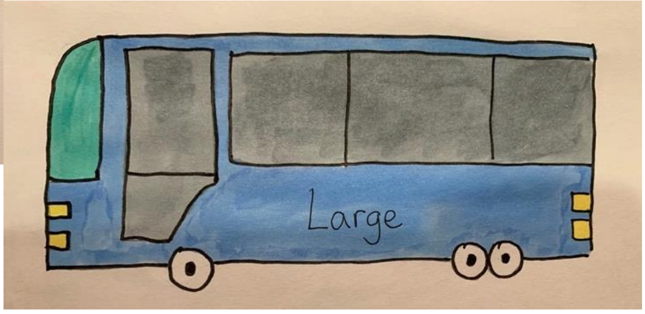
Z=6x8+20x1=68 Z=6x10+20x0=60

"We should buy 8 white cupboards and 1 brown cupboard to maximize storage volume," Sandra said.

A week later, Sandra's school was taking ninth graders to camp. Additionally, the school needs the ninth graders to provide advice on how to buy buses to transport the maximum number of students.

"The school plans to buy two types of buses to take students to camp. A small bus can carry 15 people, costs 1000 pounds, and needs 100 pounds to insure students. A large bus can carry 30 people, costs 2500 pounds, and needs 50 pounds to insure students. However, the school has only 20000 pounds to buy buses and 1000 pounds to insure students. How many buses of each type should the school buy to maximize the number of students that can be transported?" the teacher asked. "Students, it is a linear programming problem."

"I will try it. I like to solve such problems," Sandra said.



1. Identify the variables: x and y.

x: number of small buses purchased

y: number of large buses purchased

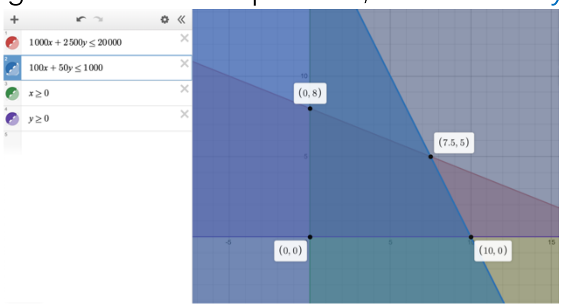
2. Identify the constraints.

A small bus costs 1000 pounds, a large bus costs 2500 pounds, cannot spend more than 20000. This is expressed mathematically as $1000x+2500y \leqslant 20000$.

A small bus needs 100 pounds of insurance, a large bus needs 50 pounds of insurance, cannot spend more than 1000. This is expressed mathematically as $100x+50y \leqslant 1000$.

Also, both x and y are greater than or equal to 0, so $x \geqslant 0$ and $y \geqslant 0$.

3. Graph the system of inequalities.



4. Write an objective function to test the corner points of the feasible region to find the optimal solution.

The number of students that can be transported will be the objective function: $Z=15x+30y$. (Find the maximum value)

Bring the 4 corner points into the objective function:

Z=15x0+30x0=0 Z=15x0+30x8=240

Z=15x8+30x5=270 Z=15x10+30x0=150

"The school should buy 8 small buses and 5 large buses to maximize the number of students that can be transported," Sandra said.

"Well done!" the teacher said. "I am sure you have learned quite well in linear programming!"

Sandra, who is in grade nine,
will use linear programming to
solve 4 problems in real life!