THE 2023 YOUNG MATHEMATICAL STORY AUTHOR (YMSA) COMPETITION

THE CINDY NEUSCHWANDER AWARD
(THE 12-15 YEARS OLD CATEGORY)

SHORTLISTED

'Whale's Oceania Odyssey' by Gisele Chong (14 years old)
at Macleans College (New Zealand)

You can read the author's inspiration for the story and the judges' comments
on:

www.mathsthroughstories.org/ymsa2023

#YMSAMaths

Whale's Oceania Odyssey

In the midst of a far-flung realm of Oceania, there was once a young blue whale and his master. He had been rescued from the captivity of whale hunters at birth. "What beautiful patterns you have," the girl breathed as she stroked the cetacean's back. "I shall name you Azure." And ever since, Azure's existence was nurtured and watched over by his devoted human companion. Over time, the little whale's heart filled with warmth. He dearly loved his wise master, who reciprocated that love in equal measure. She had taught him many things. That this place was not where he belonged and was somewhere in the glistening blue. That the ocean was filled with many wonders like dolphins and turtles, lagoons and coral reefs, underwater palaces and schools. And most importantly, Maths on a map, to find his way home. However, the bittersweet reality began to manifest, Azure had found that he was ... not so little anymore. As much as he yearned to stay with his human family, it became increasingly clear that it was no longer tenable. His master was no longer a girl, but a young woman ready to enter human society and begin her own life. With a heavy heart, the decision was made to release Azure back into the cerulean ocean, where he could flourish in his natural habitat.

As Azure was released into the colossal expanse of the ocean, he was inundated with an all-encompassed sense of liberation. The world around him was a canvas of vibrant colors and sounds, filled with the symphony of life in all its wondrous forms. Moon jellyfish bobbed lazily and rainbow fish darted here and there. He felt fully alive for the first time in his life.

Azure began to swim with purpose and vigor, eager to explore the mysteries of the underwater world. However, despite the newness and novelty of his surroundings, Azure couldn't shake the feeling of emptiness that crept into his soul. He yearned for the familiar presence of his master and his old life, and a sense of longing overwhelmed him. But deep down, the leviathan knew that his journey had just started, and his destiny lay in discovering his own kind and reuniting with his family. Azure resolved to continue on his path, hoping that one day, he would find his place in the world and fulfill his purpose.

It had been a week, and amidst the calm waves, Azure felt lost and alone. It was not until fate shone upon him when he imagined his master as a constellation in the sky pointing to something else in the sky. Azure had a flashback from his lessons in trigonometry. 'Always remember,' his master smiled kindly. "SOH CAH TOA, swim by the North Star and find a friend of mine, Wisdom of The Deep." "Wisdom knows everything about the ocean, ask him and he will show you the way back home."

In order to understand SOH CAH TOA, Azure had to understand what a right-angled triangle on a map was and had to be familiar with the concept of the Pythagoras theorem and bearings.
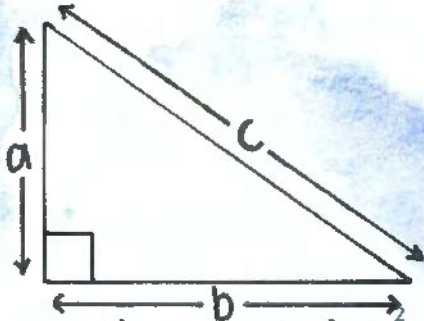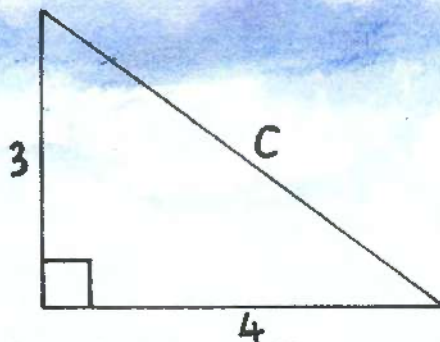

The Pythagoras theorem states that $c^2 = a^2 + b^2$,
where c is always the hypotenuse (the longest side in the triangle) and a and b are any of the two of the perpendicular sides.

This formula can help Azure find the length of an unknown distance on a triangle when the values of the other two sides are known on the map.
Here is an example Azure was shown,

The lengths of the height and base are respectively 3 and 4, so to find the hypotenuse, we simply substitute these values into the equation,

$$c^2 = 3^2 + 4^2$$
$$= 9 + 16$$
$$= 25$$
$$c = \sqrt{25}$$
$$= 5$$

Bearings is the angle measured in degrees from North in a clockwise direction. They must always have 3 digits and are always rounded to 1 decimal place if the angle is not a whole number. Here is a diagram of all the common bearings in geography,
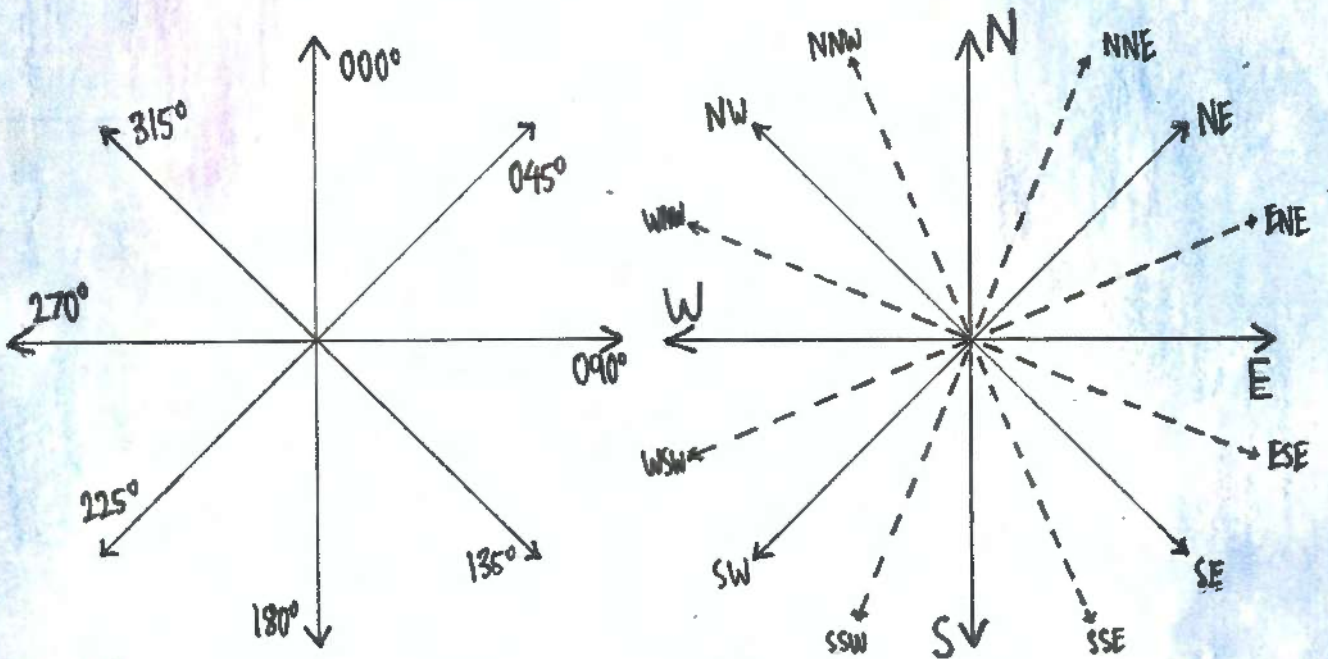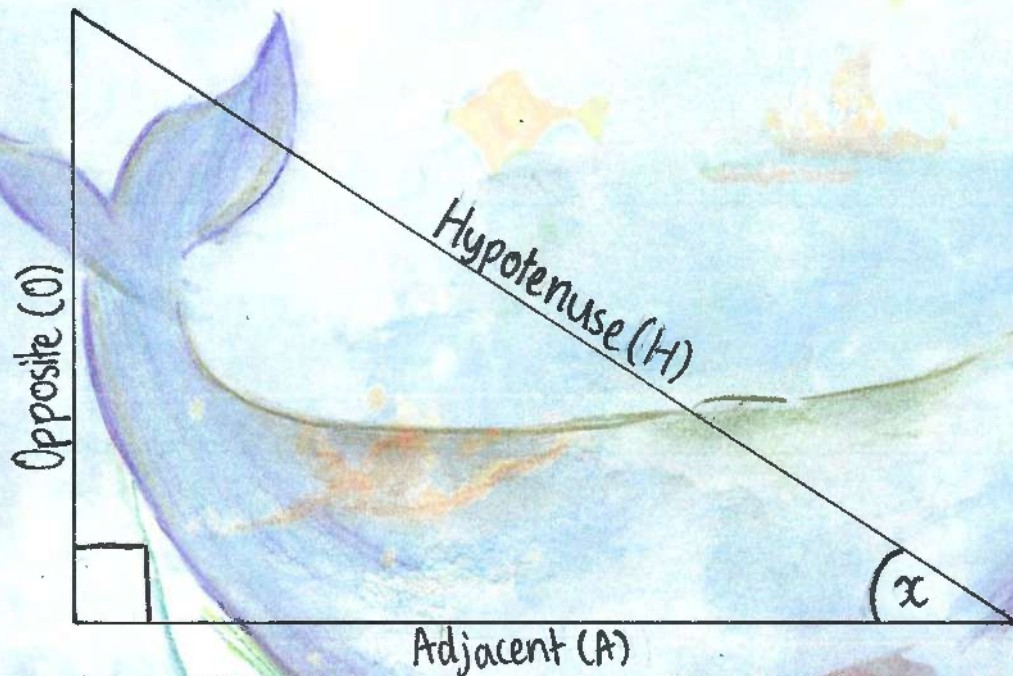

Examples

55° from North in a clockwise direction is a bearing of 055° (always 3 digits)

17.638...° from West in an anticlockwise direction is a bearing of 252.4° (always round to 1 d.p.)

129° from North in a clockwise direction is a bearing of 129°

SOH CAH TOA are the trigonometric ratios that are used to find the sides and angles in a right-angled triangle.

Opposite (O)

Hypotenuse (H)

x

Adjacent (A)

When you label a right-angled triangle, we indicate the right angle with a 'square' and the sides with:

- Hypotenuse (H): the longest side in a triangle and is opposite to the right angle
- Adjacent (A): the side between the known/unknown angle and the right angle
- Opposite (O): the side opposite the known/unknown angle (not the right angle)

And so the the trigonomic ratios are:

- SOH: $sin(x) = \frac{O}{H}$
- CAH: $cos(x) = \frac{A}{H}$
- TOA: $tan(x) = \frac{O}{A}$

Azure swims towards the North Star, shining true and bright against the melting sky. He wants to know how far he must travel to reach The Deep.

He knows the following information,

- Azure estimates he is **100 km** from where the North Star.
- The bearing from Azure to the North Star is **060°**
- He must travel **East** in a straight path

So this is what Azure imagines:
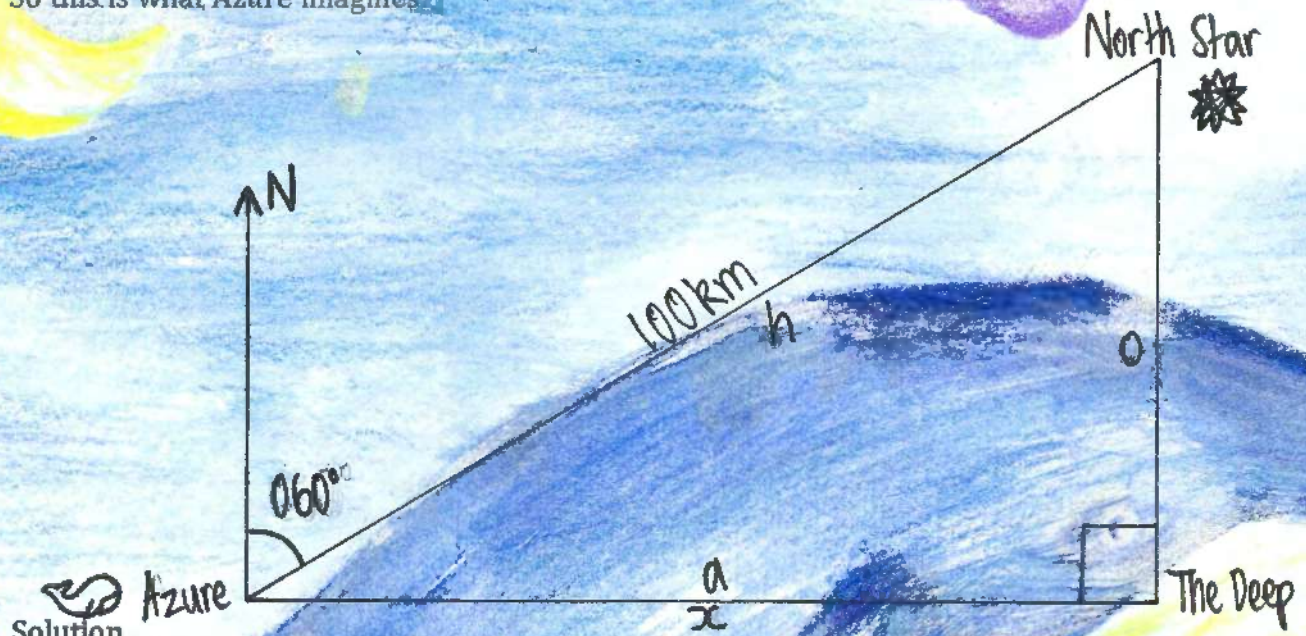
Solution

Let the distance from Azure to The Deep be x,
Azure is given the length of the hypotenuse and the angle and we want to find the adjacent x.

So he uses **CAH**, $cos(30°) = x/100$
And rearranges the equation to solve for x,
$x = 100 \times cos(30°)$
$= 50\sqrt{3}$
$= 86.6$ km (to 3s.f)

Therefore, Azure must travel **86.6 km (to 3s.f) East** to reach The Deep from his current location.

Azure had calculated his position and set a determined course, driven by an unyielding desire to return to where he belonged.

Hours stretched into days as Azure made his way to The Deep. On the way, he encountered a pod of distressed dolphins who had lost their way. "Pray to the King of the Sea!" they wailed. "We forgot the directions back home! What are we to do now!" Though the journey had already been long and perilous, Azure didn't hesitate to offer his help. He used his expertise in trigonometry to calculate the angle they needed to swim at:

The dolphins tell Azure what they remembered,

- The pod had traveled **12 km West** from their Lagoon and then **turned exactly South** at the Coral Reefs and swam for a further **10 km**
- They know they must travel back to the Lagoon in a **Northeast** direction in a straight path

So this is what Azure imagines:
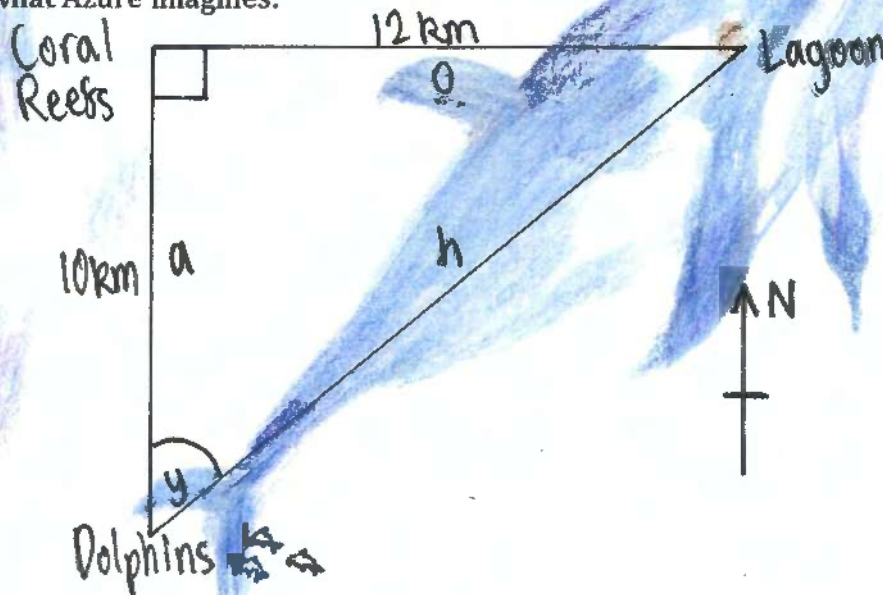
Solution

Let the unknown angle be y,

Azure is given the opposite and adjacent and we want to find the value of angle y.

So he uses TOA, $tan(y) = 12/10$

And rearranges the equation to solve for x (note the we use tan^{-1}, cos^{-1}, and sin^{-1}, when you find angles

$y = tan^{-1}(\frac{12}{10})$

$= 50.2°$ (to 3s.f.)

Since the angle is already from North at a clockwise direction, the bearing is 050.2°

Therefore, the dolphins must travel at a **bearing of 50.2° (to 3s.f.)**.

"Thank you traveler!" The dolphins waved happily goodbye after they had rejoiced in circles. "Hope you find what you're looking for!" they whistled as they dived deep down back to their lagoon.

After another week on Azure's journey, he met a confused turtle. "Oh dear oh dear..." the turtle murmured. "Sir, are you lost?" the blue whale asked. "Yes, I can't find my way back to my classmates," the turtle replied. "I think I wandered too far from the Underwater Palace, I need to go back to the school." Azure calculates the minimum distance the turtle needed to swim to reach Sea School:

Turtle tells the following information to the blue whale,

- Turtle estimates that he had wandered **140 m** away from the Underwater Palace
- Turtle knows that his school is **directly South** of the Underwater Palace
- Turtle says that the bearing of their current position from the Underwater Palace is **240°**
- Turtle knows that he must travel **East** to reach Sea School

So this is what Azure imagines:

Solution
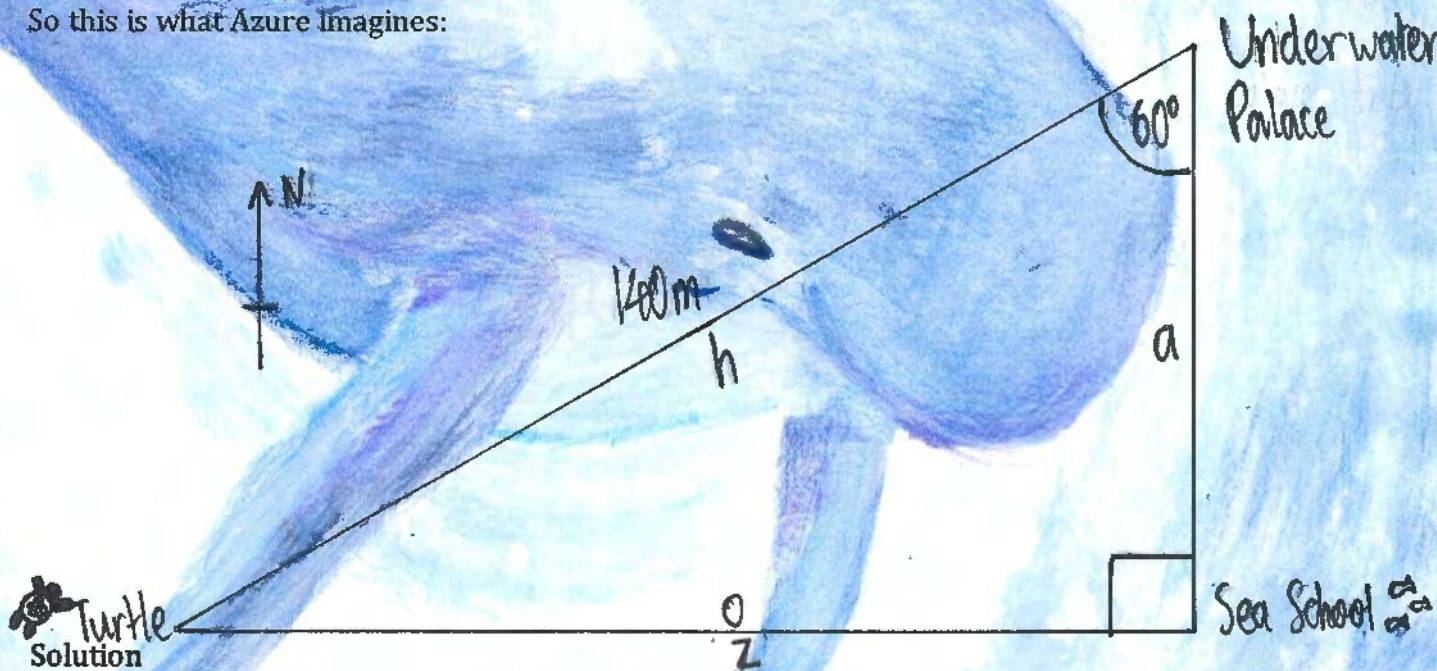
Let the distance from Turtle to Sea School be z,
Azure is given the hypotenuse and the angle and we want to find the opposite z.

So he uses SOH, $sin(60°) = \frac{z}{140}$

And rearranges the equation to solve for z,

$z = 140 \times sin(60°)$

$= 70\sqrt{3}$

$= 121$ m (to 3s.f.)

Therefore, Turtle must travel **121 m East** to reach Sea School.

"Thank you dear child...thank you" the turtle rasps. "May the sea bless you" he said, swimming off.

Days later, Azure had finally arrived at The Deep. He had found his master's friend Wisdom, a sagacious old whale whose wisdom revealed the secrets of the ocean. "You have come," Wisdom rumbles. "I have been expecting you". "I will share with you the way back Home," the wise whale said. "Listen carefully":

Wisdom tells Azure this,

- Azure is now at the at this point of the ocean, the Deep, and wants to find the **shortest distance and direction back Home**
- The wise whale says that there is a Sunken Ship and a Lighthouse **directly West** from here in that order, the distance between the landmarks being **7.15 km (to 3s.f)** from here to the Sunken Ship and **8 km** from the Sunken Ship to the Lighthouse
- The Sunken Ship is also known to be **directly South** from Azure's home
- The distance between Home and Lighthouse is **10 km**
- Azure **has** to use the trigonometric ratios when finding the direction and distance back home in order to understand and learn the wisdoms of the ocean

So this is what Azure draws:
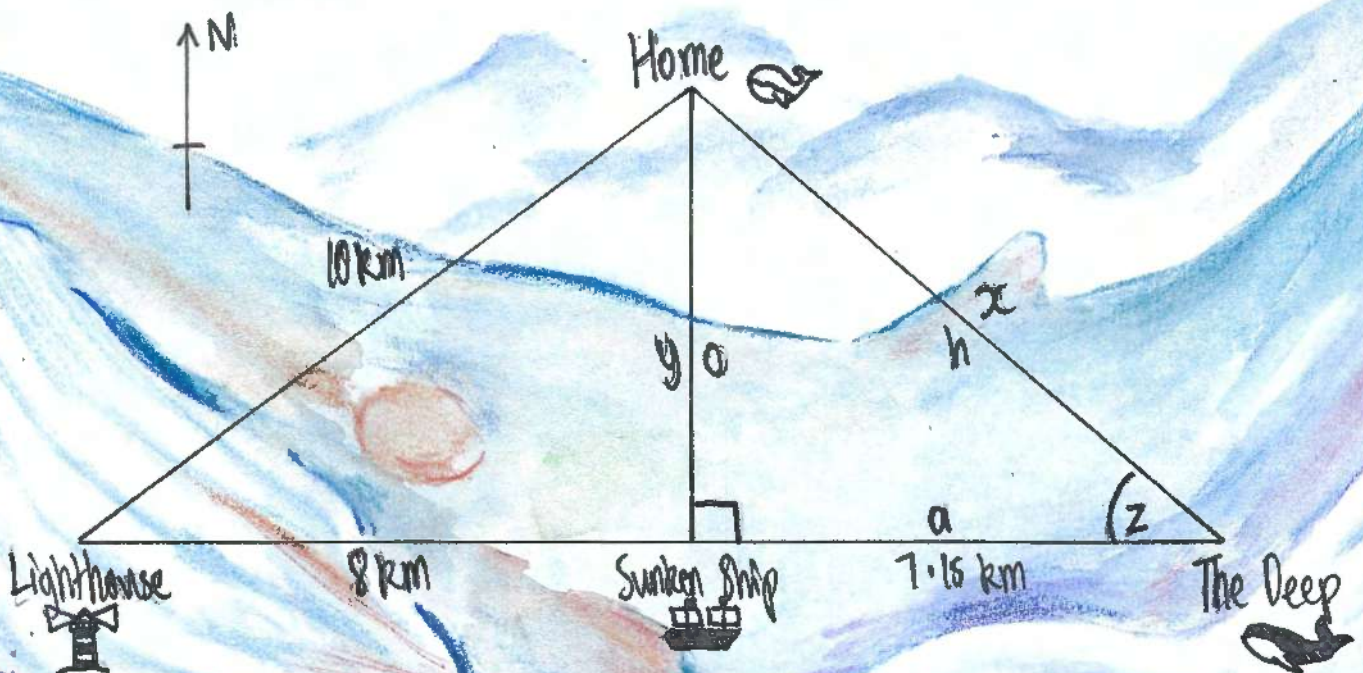
Solution

Let x distance be the distance Azure needs to travel from here to his Home, y be the distance from Home to the Sunken Ship, and z be the angle Azure needs to travel at.

10

In order to find **x** or **z** we must find the value of y by using Pythagorus ($c^2 = a^2 + b^2$),

$10^2 = y^2 + 8^2$

$y^2 = 100 - 64$

$\quad = 36$

$y = 6 \text{ km}$

Now he has the opposite and adjacent to find the unknown angle z.

So he uses TOA, $tan(z) = \frac{6}{7.15}$

And rearranges the equation to solve for z

$z = tan^{-1}(\frac{6}{7.15})$

$\quad = 40.0°$ (to 3s.f.)

Since it is 50° from North in an anticlockwise direction, the bearing would be

360°-50° = 310°

Finally, Azure has the angle (40°) and uses the opposite (6 km) to get the most accurate value of x,
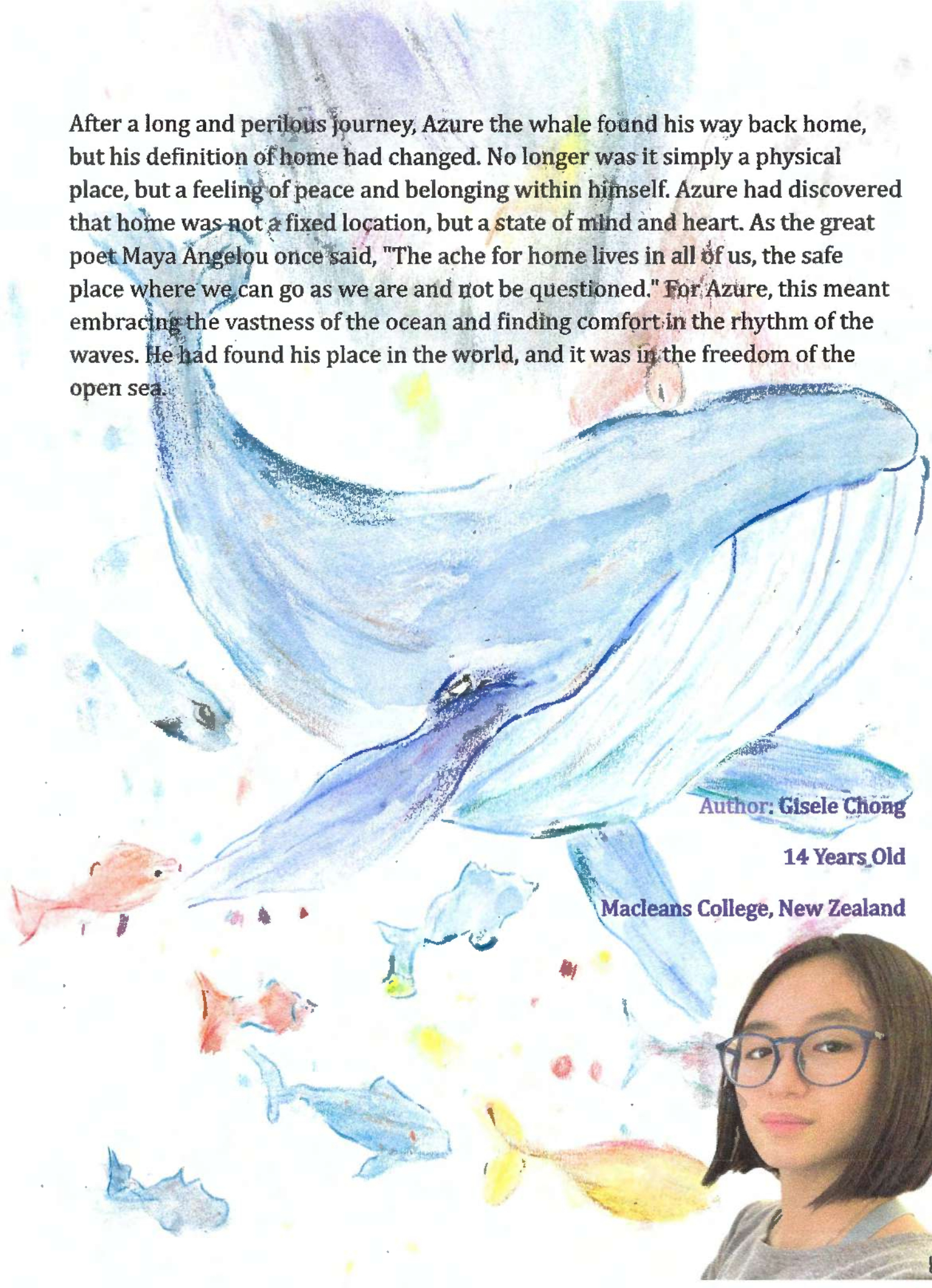
So he uses SOH, $sin(40°) = \frac{6}{x}$

And rearranges the equation to solve for x,

$x = 6/sin(40°)$

$\quad = 9.33 \text{ km}$ (to 3s.f.)

Therefore, Azure must travel at a bearing 310° from North for 9.33 km to reach Home.

After a long and perilous journey, Azure the whale found his way back home, but his definition of home had changed. No longer was it simply a physical place, but a feeling of peace and belonging within himself. Azure had discovered that home was not a fixed location, but a state of mind and heart. As the great poet Maya Angelou once said, "The ache for home lives in all of us, the safe place where we can go as we are and not be questioned." For Azure, this meant embracing the vastness of the ocean and finding comfort in the rhythm of the waves. He had found his place in the world, and it was in the freedom of the open sea.

Author: Gisele Chong

14 Years Old

Macleans College, New Zealand