THE 2023 YOUNG MATHEMATICAL STORY AUTHOR (YMSA) COMPETITION

THE CINDY NEUSCHWANDER AWARD

(THE 12-15 YEARS OLD CATEGORY)

LONGLISTED

'Cleaning with Quadratics' by Ritvik Shah (14 years old)

at Fulton Science Academy (Georgia, USA)

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#YMSAMaths

Cleaning

with

Quadratics

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Long, long ago, there was a village called Transistor Town. Transistor town was a lively village filled with Technowizards. The Technowizards were kind folk who loved creating electrical gadgets and computers. During the spring and summer, Technowizards would play outside, and make good use of their whizzing machines. During the fall and winter months, it was Working Time. Technowizards labored away in their workshops creating revolutionary gadgets. However, Technowizards weren't born technical geniuses, students had to go to Tech Academy - no matter what time of the year it was. One such student was a young boy named Elektrik. What set Elektrik apart from all the other Technowizards was that he was a very tidy and clean boy who hated trash. The mere thought of trash and junk, and unorganized spaces gave him shivers. Others in his village did not mind untidiness at all, of course, they did clean up, but when it was Working Time, nothing mattered more than inventing and creating…

One December day, during a snowy, frigid blizzard, Elektrik was preparing to go to school. Usually, after eating breakfast, he would climb into his electromobile and zoom to the academy. But, today was different. All of the roads were closed, and Elektrik's electromobile had broken down due to the temperature change. Because of this, he was forced to put on a big, fluffy jacket and walk to school. As he trudged along the pavement, Elektrik started to realize something he hadn't realized before. Trash and filth were collecting on the sides of buildings and strewn over the sidewalk. He saw spare motherboards, chipped capacitors, shattered diodes, and crushed resistors. It was a mess! Elektrik started to shiver and twitch, "This is terrible," he murmured, "I must do something about this - and quick!" The whole day, Elektrik was pestered by the horrendous disarray he had seen, and he started to develop a plan to clean up. As soon as Elektrik got home, he started drawing out blueprints and diagrams to create a robot that could dispose of all the trash in the city. The plan was simple, the robot would scoop up trash with a robotic arm, compress it into a ball, and toss it into a recyclable or nonrecyclable dumpsterbot. The invention would be named "Cleanbot".

Soon enough, however, Elektrik realized a flaw in his plan…

During the tossing phase of the robot's clean-up phase, the Cleanbot would need to throw the ball of trash at different heights to ensure that it would enter the recyclable or non-recyclable bins. But, Elektrik didn't know how hard the robot would need to throw the ball. To solve the dilemma, Elektrik called his AI friend: Floatbot. Floatbot was a cheerful, floating AI robot who knew everything about Transistor Town. She was also very well-versed in math. As soon as Floatbot heard that her friend needed assistance, she whizzed over and asked,

"What happened, Elektrik, I hope everything is ok!"

"Yes, everything is fine, except for the fact that Transistor Town is a mess. I have a solution to clean up, but I don't know how to implement it…"

"Oh, well that seems terrible. Tell me, what seems to be the issue?"

"Well, I am inventing a Cleanbot that will take trash from the streets and throw it into Dumpsterbots, but I don't know how to calculate how hard the robot needs to throw the trash in order for it to fall properly into the bins."

"Hmm… That's a very interesting problem, and I know just how to solve it!"

"Really! Please teach me! I can't wait to learn!"

"Of course! First, we need to properly define the problem, because a problem half defined is a problem half solved. To do this, we need all the measurements and all the criteria. First, the Cleanbot will need to either throw the trash into recycling bins or non-recycling bins. Recycling bins are 11 feet tall, and non-recycling bins are 15 feet tall. The trash must clear a height of 11 or 15 feet so that it can properly fall into the bins. We also need to define how much time it must take for the trash to clear the height because just in case it doesn't, it is always safe to include a backup system that will throw the trash again - we don't want any random trash balls for people to trip on!"

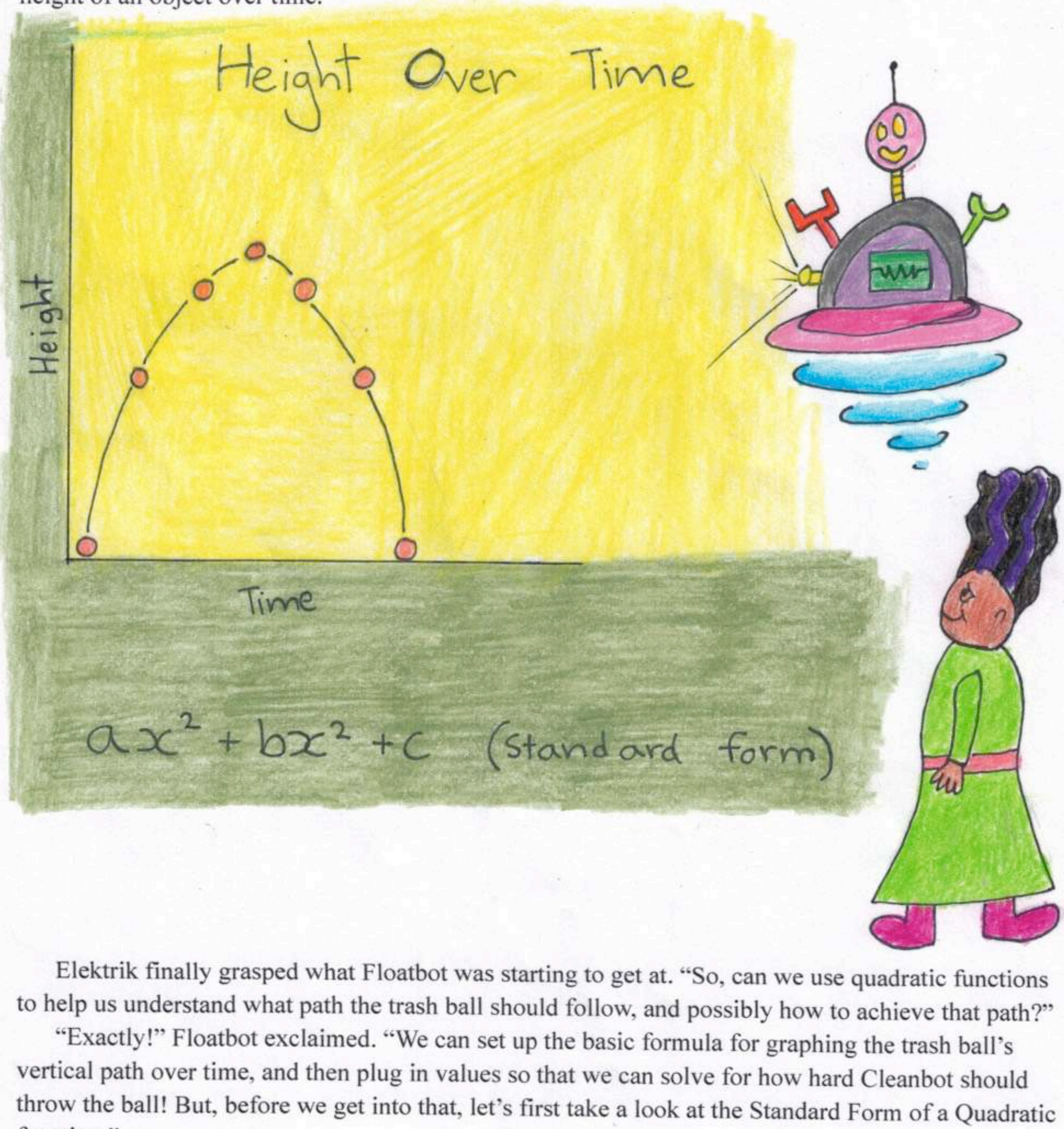
Elektrik thought for a few moments, and decided, "I think the fail-safe time should be one second. It is not too long, and not too less."

"I agree!"

"Now that we have defined the problem, we can start doing the math! For this particular issue, we need to use the quadratic gravity formula."

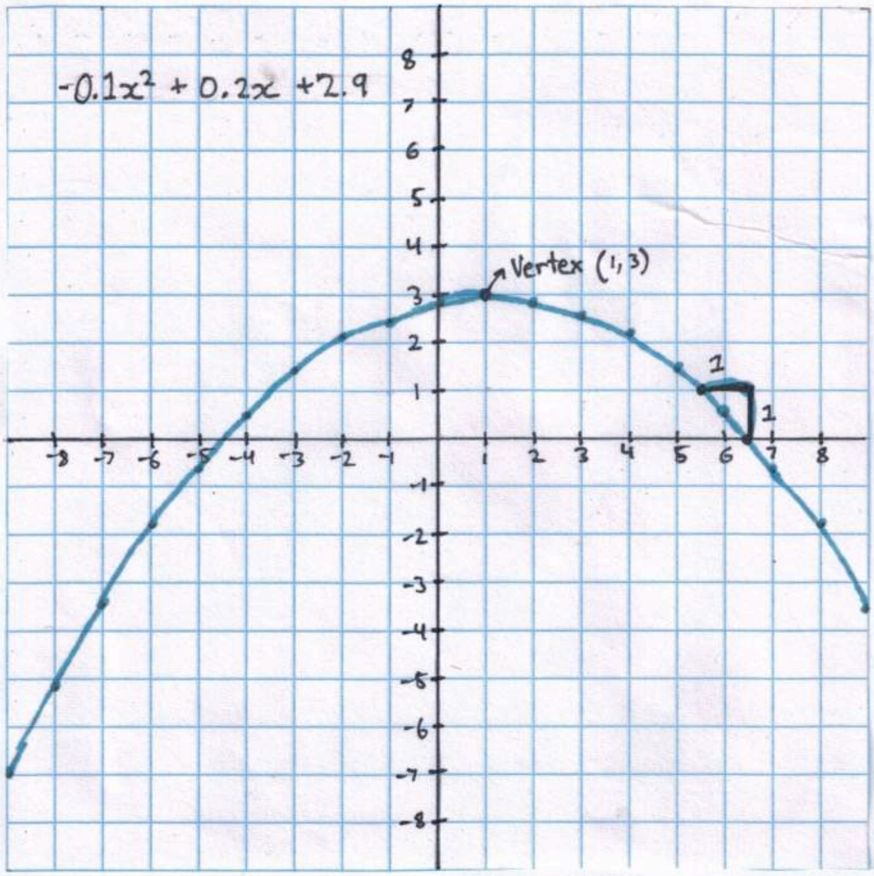
Elektrik was very confused, "What's the 'Quadratic gravity formula?' And what's a 'Quadratic?' And why 'Quadratic?'"

"Whoa, hold your horses! It's good that you're asking questions, but one at a time. You see, all objects that are thrown follow a specific vertical path over time. That path forms a curve called a parabola. Parabolas can be graphed by special functions called quadratics. A quadratic function is any function in which the relationship has a degree of two. This means that the highest exponent in the function is two. Quadratics can be in any form but are usually arranged in standard form for simplicity. The standard form of a quadratic is $ax^2 + bx + c$. Physicists have found the proper constants and have understood where to place certain parameters to correctly graph the vertical height of an object over time."



Elektrik finally grasped what Floatbot was starting to get at. "So, can we use quadratic functions to help us understand what path the trash ball should follow, and possibly how to achieve that path?"

"Exactly!" Floatbot exclaimed. "We can set up the basic formula for graphing the trash ball's vertical path over time, and then plug in values so that we can solve for how hard Cleanbot should throw the ball! But, before we get into that, let's first take a look at the Standard Form of a Quadratic function."



x	y
-7	-3.4
-6	-1.9
-5	-0.6
-4	0.5
-3	1.4
-2	2.1
-1	2.6
0	2.9
1	3
2	2.9
3	2.6
4	2.1
5	1.4
6	0.5
7	-0.6
8	-1.9
9	-3.4

"In Standard Form, the coefficients *a*, *b*, and *c* are transforming and translating a parent function x^2. The coefficient *a* controls the narrowness of the graphed line, the farther away *a* is from 0, the narrower the graph. If *a* is negative, then the graphed line will 'point' downward. *B*, as a coefficient to x, when added to ax^2, controls the positioning of the vertex (the turning point of the parabola) in relation to the y-axis. The coefficient *c* is the y-intercept of the graph, or the point at which x is equal to 0."

Elektrik was amazed, "So all of the numbers and letters are just changing what x^2 looks like, and the way *x* inputs relate to *y* outputs?"

"Yes! Spot on!" Delighted that Elektrik was starting to understand basic quadratics, Floatboat decided to introduce a more complex idea: "Now that you understand that, I would like to ask you a question. Do you believe that the graph I just showed you correctly relates vertical positioning in feet (y-axis), and time in seconds (x-axis) of a falling object?"

Elektrik considered this for a moment, and studied the graph Floatbot had hologrammed onto the wall: "Well, the graph shows an object falling just one foot in one second at its steepest before reaching a vertical position of zero feet - which would just be the ground… So no. I don't think that this graph correctly relates vertical positioning and time."

"Correct again! So what should we change about *a* so that it is more accurate?"

Elektrik thought, and replied, "The value of *a* should be much farther from 0."

"Yes, you are exactly right! Physicists have found the exact value, which appropriately graphs the force of gravity acting on an object. This number is -16. Now, what do you think *c* and *b* should be?"

"Well, other than gravity, there is no other natural force acting on a falling object, so maybe the force with which we throw the ball? But why would we need two coefficients for that?"

"You're very close. Yes, you are right that one of the coefficients is assigned the value of the initial velocity we give to the trash ball. The other though, is the starting height of the ball. The initial velocity is b and the initial height is c."

All of the knowledge Elektrik was absorbing was starting to jumble up. Cs, Bs, As, Xs, so many letters and coefficients and variables. "Floatbot, can't we name the coefficients and variables something else? It's all so confusing!"

"Elektrik, I am glad you asked. I was just about to get to that. You see, for any function, equation, or expression, you can always change what you name variables, so, for our convenience, we can rename every a, b, c, x, and y. Because a represents the downwards force of gravity, we can name it g. Because b is the initial velocity, we can rename it v_0. Because c is the initial height, it would make sense to name it h_0. And, because our x-axis is being used to count time, let's switch x with t. Our new function definition is:

$$f(t) = gt^2 + v_0t + h_0$$

"Oh! Floatbot!" Elektrik exclaimed, "I know what else we can plug in, one second for t, and, of course, -16 for g!"

"Good! Because we want to understand how hard we should throw the ball based on the time constraint of one second:"

$$f(1) = -16(1) + v_0(1) + h_0$$
$$f(1) = -16 + v_0(1) + h_0$$

Elektrik suddenly realized another value they already had to plug in, "Floatbot, we also know the initial height! My robot, when throwing, releases the ball at a height of three feet! Therefore, h_0 would be three!"

"Exactly! You are a very fast learner, Elektrik! In addition to plugging in three, we can also equate our relation with 11, because that is the height we are trying to achieve. After making these changes we get:"

$11 = -16 + v_0 + 3$

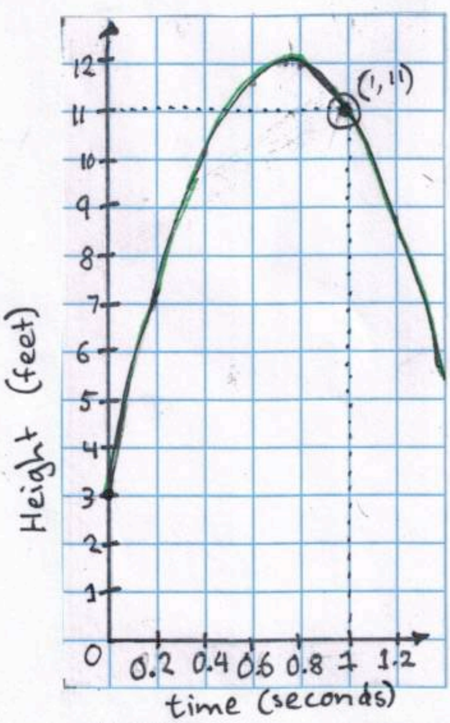
"And, now that we have only one unknown to solve for, which is the initial velocity, or how hard the ball must be thrown, we can easily simplify and find our answer!"

Elektrik already new how to simplify equations, so he enthusiastically asked Floatbot to hologram a whiteboard so he could solve the equation.

$11 = -16 + v_0 + 3$
$11 + 16 = -16 + v_0 + 3 + 16$
$27 - 3 = v_0 + 3 - 3$
$24 = v_0$

"Floatbot!" Elektrik exclaimed, "Floatbot, I have the answer, it's 24 feet per second! That's how fast my robot needs to throw the ball!"

"Very good Elektrik! I am extremely proud of you!" Floatbot congratulated his peer. "However, it's always good to check your answers, which is why we should graph the full function, $f(t) = -16t^2 + 24t + 3$, and make sure that when t is one, then y should be 11! To do this, simply make a table of values, plot the points, check to see if your solution is correct, and at the same time, have a visual aid that tells you what height the trash ball will be at every second."



x	y
0	3
0.2	7.16
0.4	10.04
0.6	11.64
0.8	11.96
1	11
1.2	8.76

$v_0 = 24!$

Elektrik quickly made a graph, and showed it to Floatbot, "I think it is right! When t is one, y is exactly 11!"

"It is heartwarming to see you progress so much in this sub-topic of math. Now, I think it's time you try doing a problem on your own. We just found the initial velocity required to throw a trash ball into a non-recycling bin, but what about a recycling bin?"

Elektrik was extremely excited to try out quadratics on his own, and he also wanted to do it right, so he followed the steps Floabot had followed while helping him solve the first problem. First, Elektrik needed to define the problem;

"For recycling bins, my robot needs to throw a ball to a height of 15 feet in one second, from an initial height of three feet. Now that I have defined my problem, I can plug in the values into a function. First, I need to write down my quadratic gravity formula."

$$f(t) = gt + v_0t + h_0$$

"Then, I can replace *t* with one:"

$$f(1) = g(1) + v_0(1) + h_0$$

"After that, it is essential that I plug -16 into g:"

$$f(1) = -16(1) + v_0(1) + h_0$$

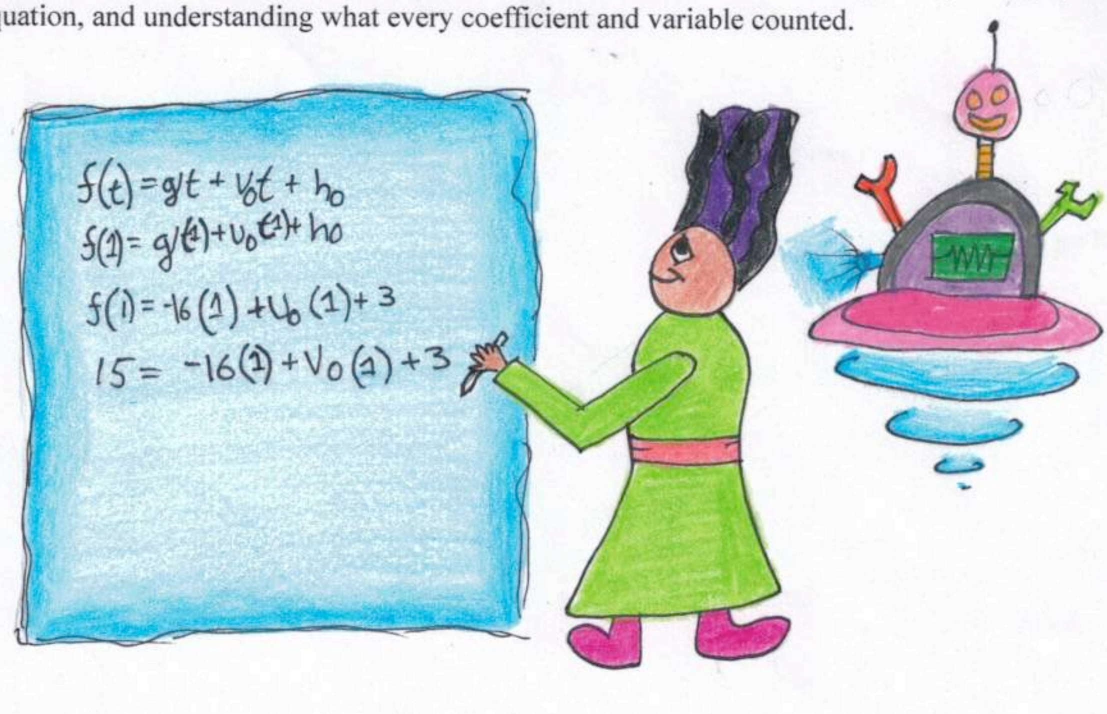
"Then, I have to plug three in to h_0 to define my initial height:"

$$f(1) = -16(1) + v_0(1) + 3$$

"And finally, I have to equivalate my relation to 15, because that is the desired height I need at the unknown initial velocity of v_0!"

$$15 = -16(1) + v_0(1) + 3$$

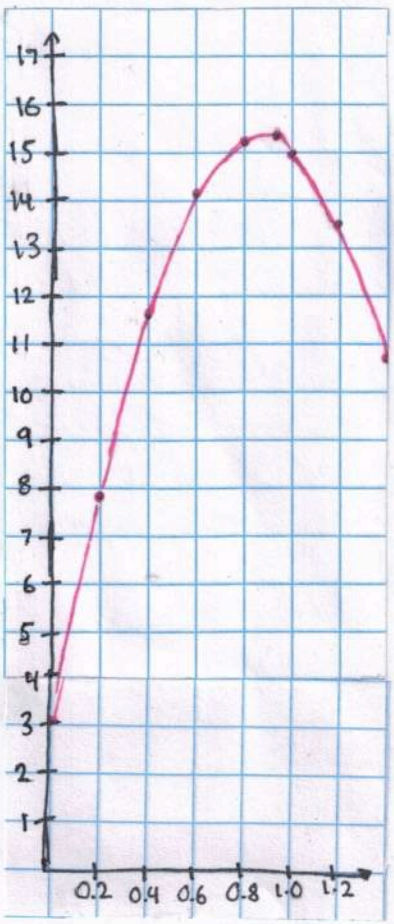
Floatbot was very impressed with Elektrik's proficiency and clarity regarding setting up an equation, and understanding what every coefficient and variable counted.

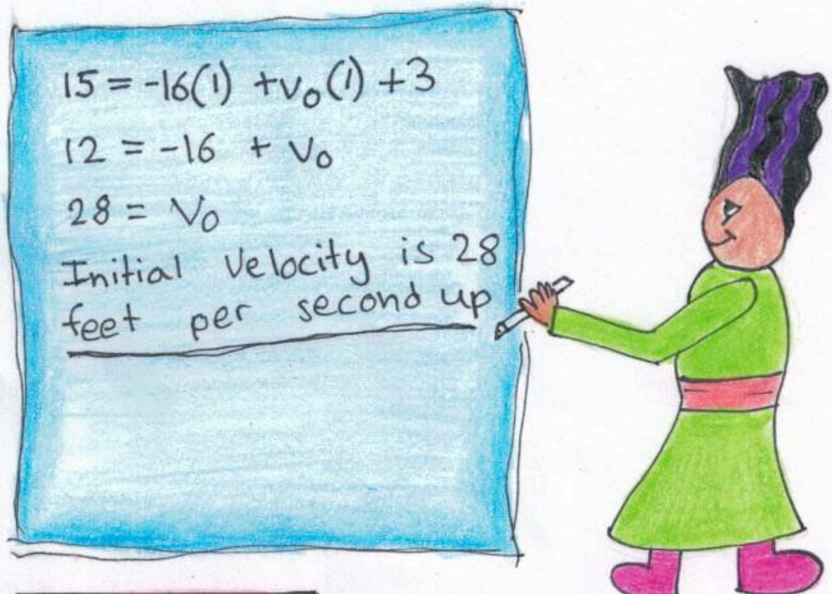


Elektrik gazed at his unsimplified equation, extremely happy that he had come this far on his own. But, he was unconfident that he was doing everything correct so far. He asked Floatbot, "Did I do it right? Until now?"

Floatbot smiled, and replied warmly, "Well, I would love to tell you, but if you keep relying on me for answers, how will you do it on your own in the future? Finish solving, and you will know."

Elektrik understood what Floatbot was saying, and realized that nothing terrible would happen if he got the question wrong, he would just check his work and solve it again. So, Elektrik put on a determined face, and began simplifying the equation.





x	y
0	3
0.2	7.96
0.4	11.64
0.6	14.04
0.8	15.16
1	15
1.2	13.56

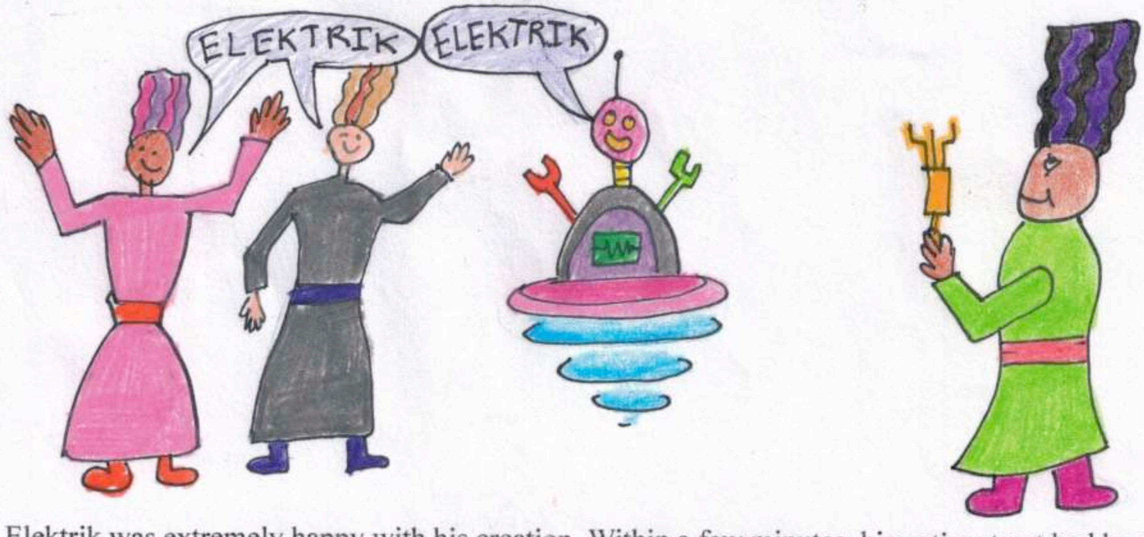
"Floatbot! I found the solution, and it's correct! The table of values, and the Graph confirm my solution," Elektrik exclaimed.

"Very good Elektrik, you have just used Math to solve a complex, and real world problem."

Elektrik blushed, "I would never have been able to do it without your guidance. Thank you so much for teaching me quadratics! What can ever do to repay you?"

"Oh Elektrik, you don't need to 'repay' me! This was the least I could do. You were trying to make our community a better place, and I was more than happy to pitch in. Now, it would really make me happy if you programmed your Cleanbot to throw the trash balls with the right amounts of force - I would love to see it in action!"

Elektrik smiled, and enthusiastically programmed and finished his Cleanbot. He then took it outside, set it on the ground, and turned it on. The robot immediately whirred to life. It rotated it's head, and moved around, before unfolding it's arm. Then, it stopped for a few seconds, and Elektrik's heart sank. But then, all of a sudden, it drove over to a mess of trash. First, it pushed all the trash into one place. Then, it used it's arm to sort and shape the trash pile into separate trash balls. The trash balls were organized into two different sub-piles; one for recycling, and one for non-recycling. Then, it picked each non-recyclable trash ball, one by one, and moved them over to a non-recycling trash bin. It one by one started throwing them into the bin, and every ball landed in the bin perfectly in time. After that, the Cleanbot picked up the recyclable trash balls, and threw them into the recycling bin - perfectly in time.



Elektrik was extremely happy with his creation. Within a few minutes, his entire street had been completely cleaned up! Other Electrowizards came outside to see what was happening, and were surprised to see their roads so clear and tidy. Everyone cheered Elektrik's name, and he was regarded a hero. Elektrik was even awarded the Transistor of Honor for his helpful contribution to the city. His name became the motto for robots - like an adjective to describe helpful, mechanical components that relied on electricity to work. After all, all robots are *electric*.

Cleaning With Quadratics

One winter day, a young boy named Elektrik is forced to walk to school instead of riding in his electromobile. While walking, he sees the trash and untidiness of his city. Follow Elektrik's journey, as he uses math and quadratics to clean up his home!