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LONGLISTED

'On the Way' by Sunny Wang (13 years old)
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You can read the author's inspiration for the story and the judges' comments on:

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#YMSAMaths

On The Way

Similar Triangles
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On a vast prairie lived many animals. They lived a peaceful life of their own; until one day, they were awakened by some new and loud noises. The white storks were the first to notice, and one of them flew toward the direction of the noise, trying to find the source. The stork came back with horrible news: the humans with their terrifying machines were approaching their home. After the white storks received this news, they quickly sent a few of them to inform the other animals. Soon, this news spread all around.

despite the animals' general habit of staying away from each other, now gathering all together. One of the elephants broke the silence,

"It is clear that we only have one choice. I know this is very crucial, but I'm afraid we have to migrate to find another home."

There was a long silence, then one of the antelopes stepped forward.

"This is so ridiculous! It is the humans who steal our homes and environments, but it's us who have to deal with our loss!"

The zebras joined in, "Yes it is not fair! We should resist the humans!"

All the animals started complaining and shouting.

The old yak stomped the ground for everyone to quiet down—the oldest of all the animals, receiving the most respect.

"I agree with the elephants, brethren. We all know the outcome if we fight against the humans. It is best for us to avoid them as much as possible. It is time to leave our home."

So, the animals set out on a journey to find a new home away from all the infrastructure.

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However, along the journey, a thick river trapped them. Based on the two stakes and single broken rope next to the running current, they could ascertain that there once was a bridge to cross the river, but it broke. Some of the animals tried to cross the river, but the water was too fast, forcing them to return to shore.

The antelopes looked around, suggesting, "we could use these woods to make a bridge to go over the river."

The animals, all assenting to this idea, sent the storks to first fly over the river and investigate the situation on the other side. Soon, the storks returned.

"We have seen that there is nothing dangerous across the river, but there is a long canal along the river bank."

"This must be one of the man-made infrastructure projects," said one of the elder elephants, "If we cut a tree that is too tall, we will destroy this canal. I believe that everyone understands how depressing it is for our own belongings and homes to be destroyed, but just because the humans have conducted such actions upon us, it does not mean that we should do the same to them."

Even though all the animals were very dissatisfied about how the humans had affected them, after the discussion, they still decided to accurately measure the width of the river to avoid destroying the canal.

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With the day nearly aproaching evening, all knew they would have to be quick, lest they would not be able to establish a safe resting place for the night. Now, they had to measure the width of the river in order to choose the correct tree to cut down. However, a problem emerged: the animals did not have a suitable measuring tool, because the width of the river was too long, and the rulers the animals had were all far too small Just when everyone was stuck, an inconspicuous cuckoo bird spoke,
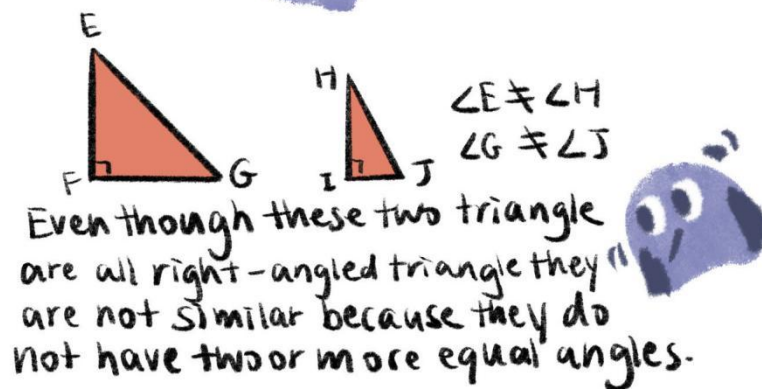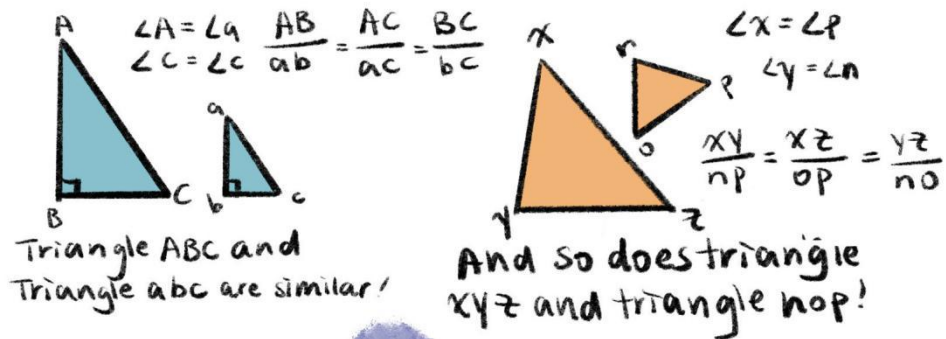
"I have a method to find the width of the lake, we can use similar triangles."

"Uh....What is 'similar triangles?'" the other animals asked, "Indeed! What do triangles have to do with getting the length of the river?"

The cuckoo began to explain: "The so-called 'similar triangles' are the same in shape, but different in size. However, as long as their shape is the same, they are similar no matter how their size changes; thus, they are called 'similar triangles.'"

"But how do you know if two triangles are the same in shape or not?" interjected a youthfully pretentious zebra.

"Granted that two triangles are similar, that their three inner angles will be the same, and the three sides will all possess the same ratio, is epistemically certain."

$\angle A = \angle a \quad \dfrac{AB}{ab} = \dfrac{AC}{ac} = \dfrac{BC}{bc}$
$\angle C = \angle c$

Triangle ABC and Triangle abc are similar!

$\angle x = \angle p$
$\angle y = \angle n$

$\dfrac{xy}{np} = \dfrac{xz}{op} = \dfrac{yz}{no}$

And so does triangle xyz and triangle nop!

$\angle E \neq \angle H$
$\angle G \neq \angle J$

Even though these two triangle are all right-angled triangle they are not similar because they do not have two or more equal angles.

"That is all well and good, Cuckoo, but how exactly can we use this to solve the length of the river?"asked another curious cub.

The cuckoo smiled and pointed her beak at the broken rope on the other side of the river. "I need someone to help me pull that rope to our side of the river, remenber to tilt it a little bit."A stork offered to help out. As the rope is pulled tight at the animals side, it looks like this:

canal

rope
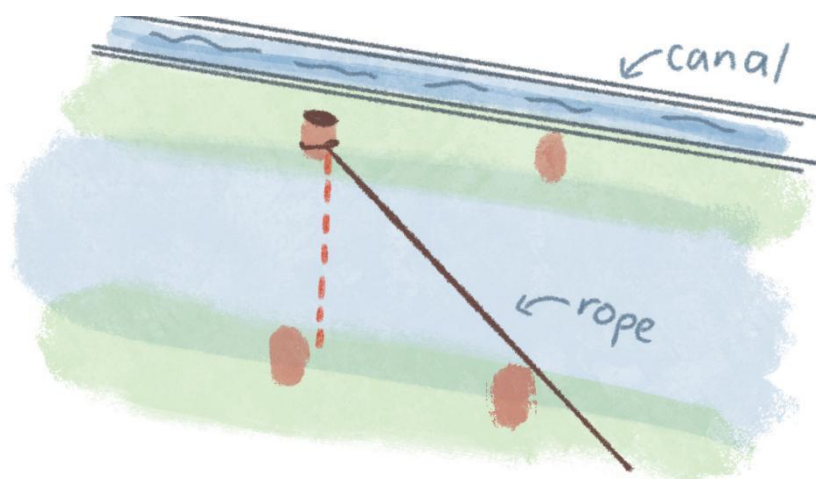
"now I am going to explain to all of you how I am going to use similar triangles to solve the width."the cuckoo said confidently.
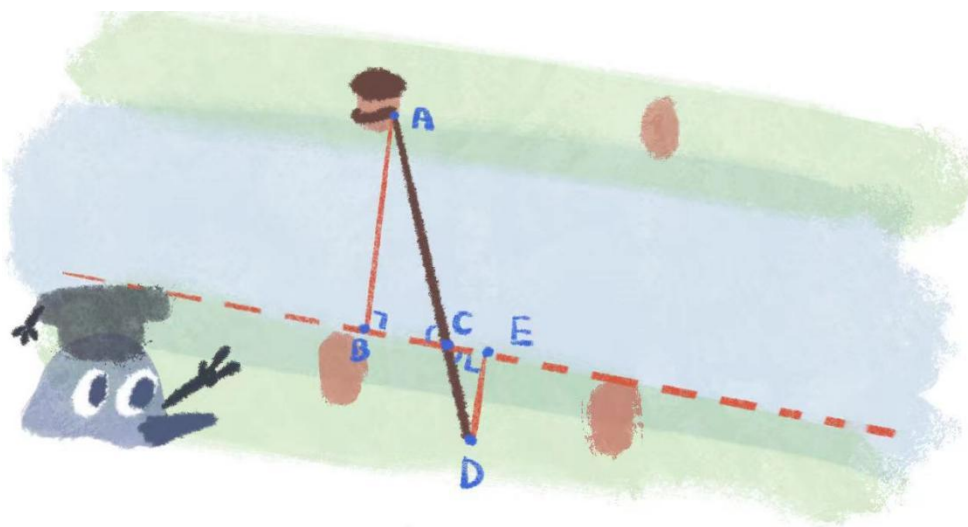
① Line AB is parallel to line DE:
$$\angle B = \angle E = 90°$$
② when two lines cross, the diagonal angles are equal:
$$\angle C_1 = \angle C_2$$
③ Two pairs of the angle are equal, therefore the last pair will also be equal:
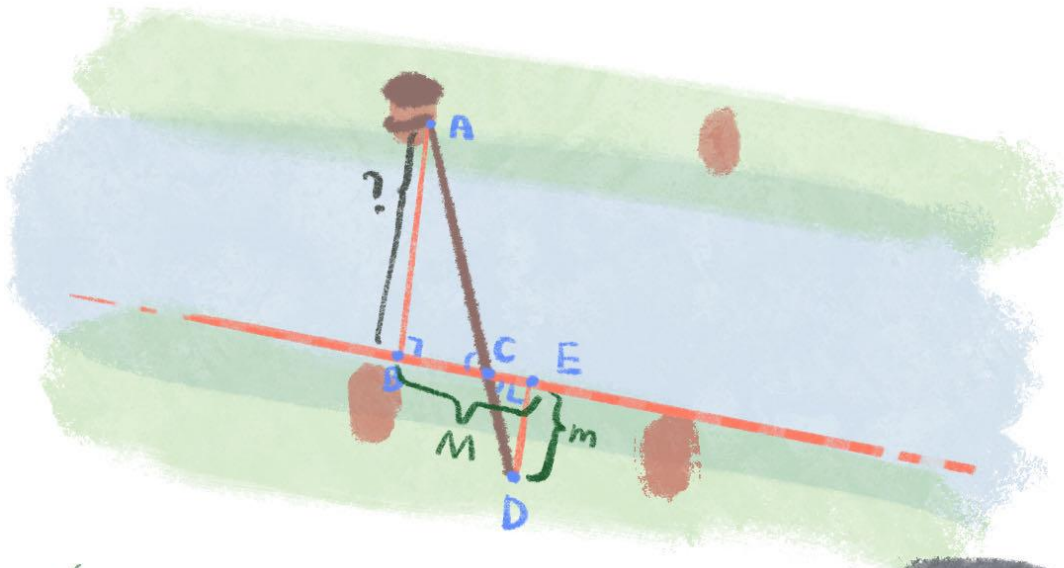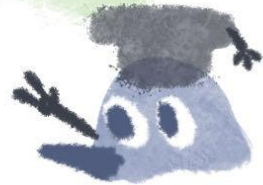$$\angle D = \angle A.$$
Conclution:
$\triangle ABC$ and $\triangle DEC$ are similar

"Now we proved that these two triangles are similar triangles, here is a property of similar triangles:

The corresponding sides of the two triangles will always be proportional...now having this property in mind, we can start to measure!"

* M and m the part we need to measure.

① After the measurement we need to find the ratio between line BC and line CE:

Measurement:
line BC = 5 meter
line CE = 1 meter.
line DE = 5 meter
Calculation:
$$\frac{BC}{CE} = \frac{AB}{DE} \qquad \frac{5}{1} = \frac{AB}{5}$$
Therefore AB = 25 meters.

Now that we have determined the distance across the river which is 25 meters, we will need to ascertain which of these trees best match our calculated number. "hmmm, but it is too hard to find the height of the trees using this method, what should we do now?"the other animals asked.

The cuckoo did not answer but looked up at the sun and down at the shadow of the trees and suddenly she looked back at the animals and exclaimed, "Yes, I have an idea! I need a stork to stand by the trees to help."

The cuckoo then started to measure, measure and measure, then calculate, calculate and calculate....


...and finally the cuckoo shouted in joy, "I've got it! This tree have a suitable height！" All of the animals were confused, they all looked doubtfully at the tree and back at the cuckoo. The cuckoo signed and said, "of course I will explain to you all. Do you all see the shadow of the stork and the tree? The shadow, the stork and the sunlight rays makes a right-angled triangle, and so does the shadow, the tree, and the sunlight..."

① the sun rays are parallel to each other:

line Na is parallel to line Zb

② The stork and the tree are both perpendicular to the ground:

line OP perpendiculars line NP

line xy perpendiculars to line zy.

③ NP and zy are bothe the floor:

line NP parallels to line ZY

?

a b

x

O

P

N

Z

Y
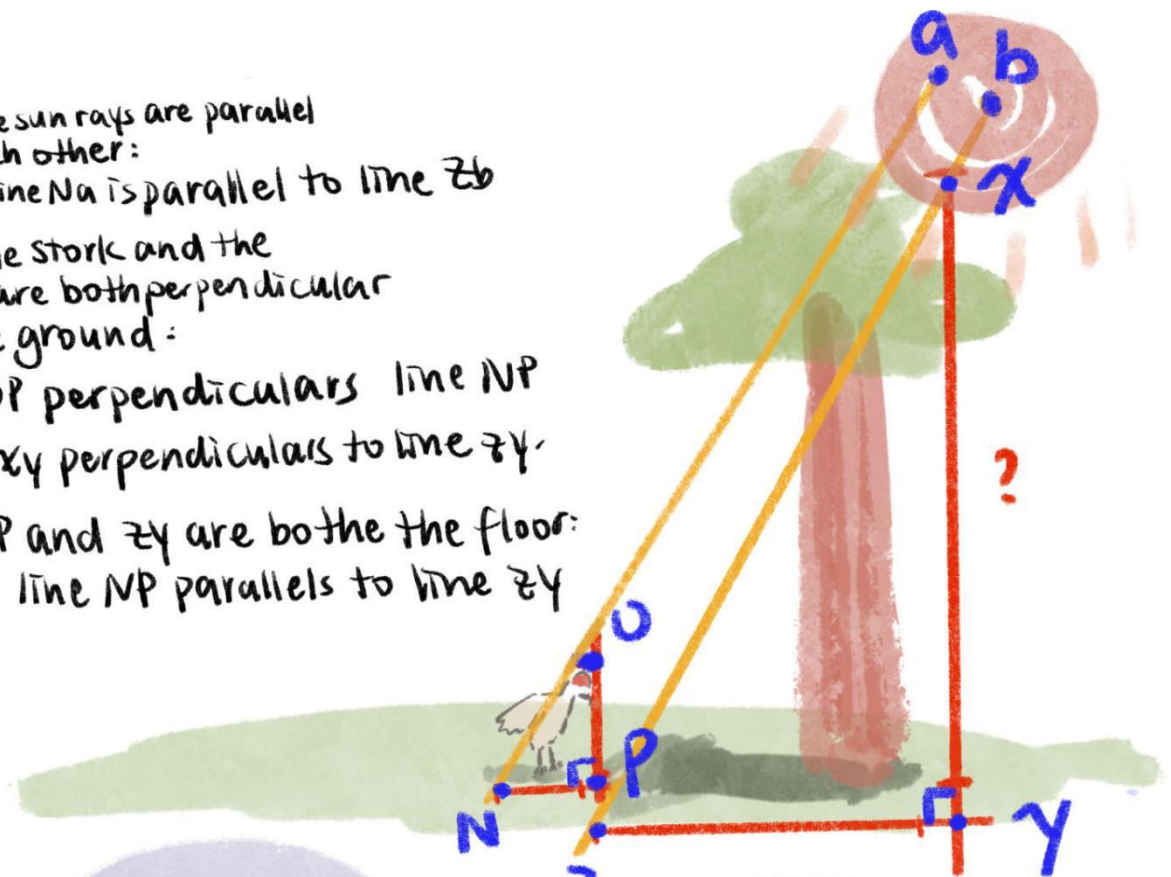
Conclution:

① ∠P = ∠Y = 90°

② ∠N = ∠Z

③ ∠O = ∠X

SO· △NOP and △Zxy are Similar.

Here are the measurements I did for the stork the tree and the shadows:

measurements:
height of stork (OP) = 1 meter
length of stork's shadow:
NP = 0.5 meters
length of tree's shadow:
ZY = 15 meters.
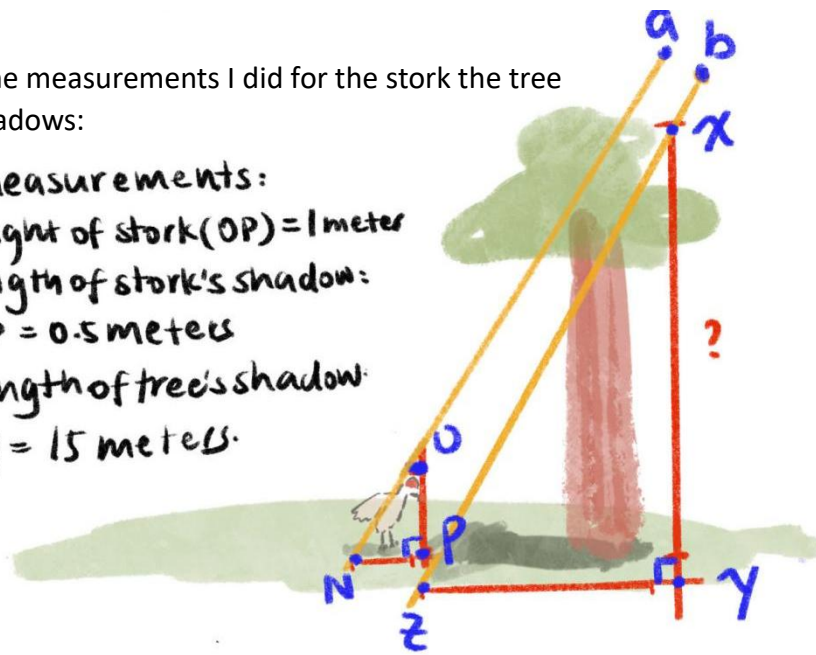
$$\frac{NP}{ZY} = \frac{OP}{XY}$$

$$\frac{15}{0.5} = \frac{1}{XY}$$

therefore XY = 30 meters

"using this method, I calculated that the height of the first two trees: 20 meters and 35 meters, which are either too short that can not make a bridge, or too long that will destroy the canal. Consequently, this third tree with a height of 30 meters is our best choice to cut down."

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The sun is almost setting and the animals had to cut the tree down quickly. With everyone involving and helping, the tree was cut down and used as a bridge just in time. Cheering and joyfulness filled the group as they crossed over the river one by one and faded into the sunset.