THE 2023 YOUNG MATHEMATICAL STORY AUTHOR (YMSA) COMPETITION

THE CINDY NEUSCHWANDER AWARD
(THE 12-15 YEARS OLD CATEGORY)

LONGLISTED

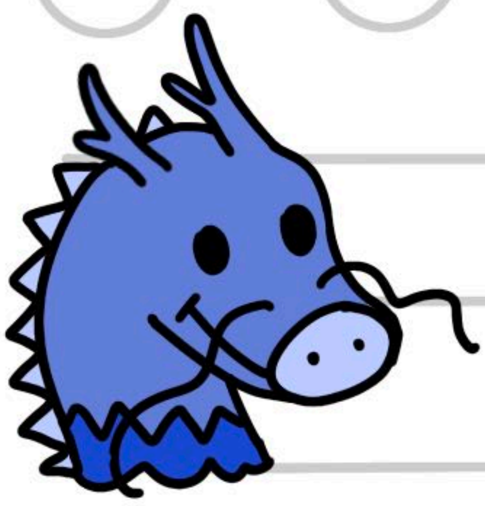
'Locker Luck' by Linda de Boer (14 years old)
at Dulwich College Beijing (China)

You can read the author’s inspiration for the story and the judges’ comments on:

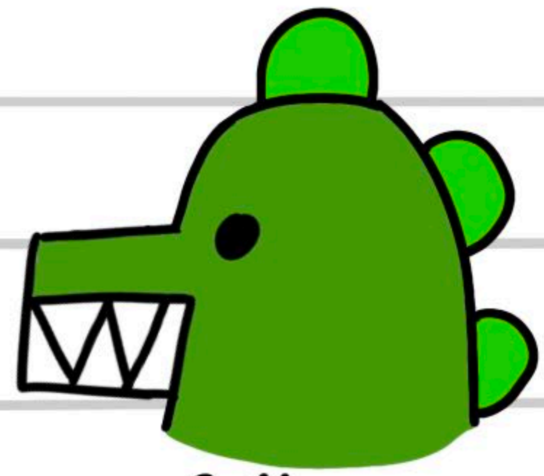
www.mathsthroughstories.org/ymsa2023

#YMSAMaths


LOCKER LUCK
Read about FACTORIALS in PROBABILITY!
By Linda de Boer Dulwich College Beijing



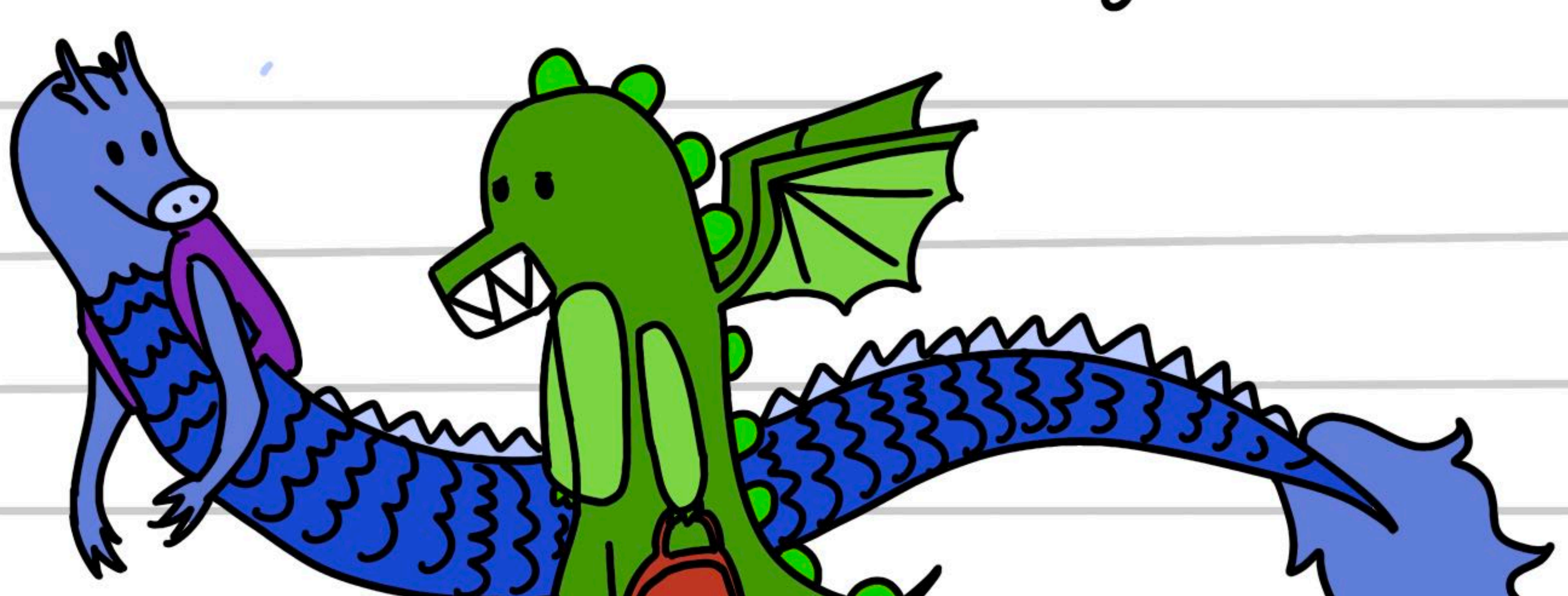
Loey



Billy

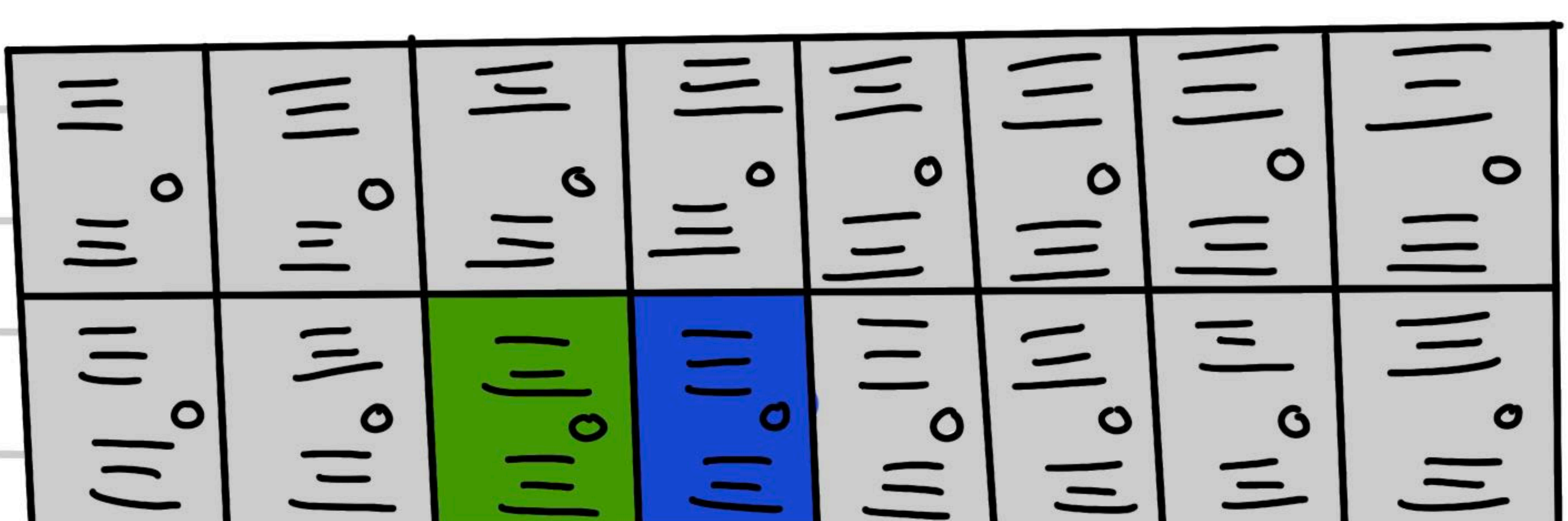
Billy and Loey are best friends. Next month, they'll be going back-to-school – and they want to work out the probability of the two of them being assigned lockers that are next to each other.

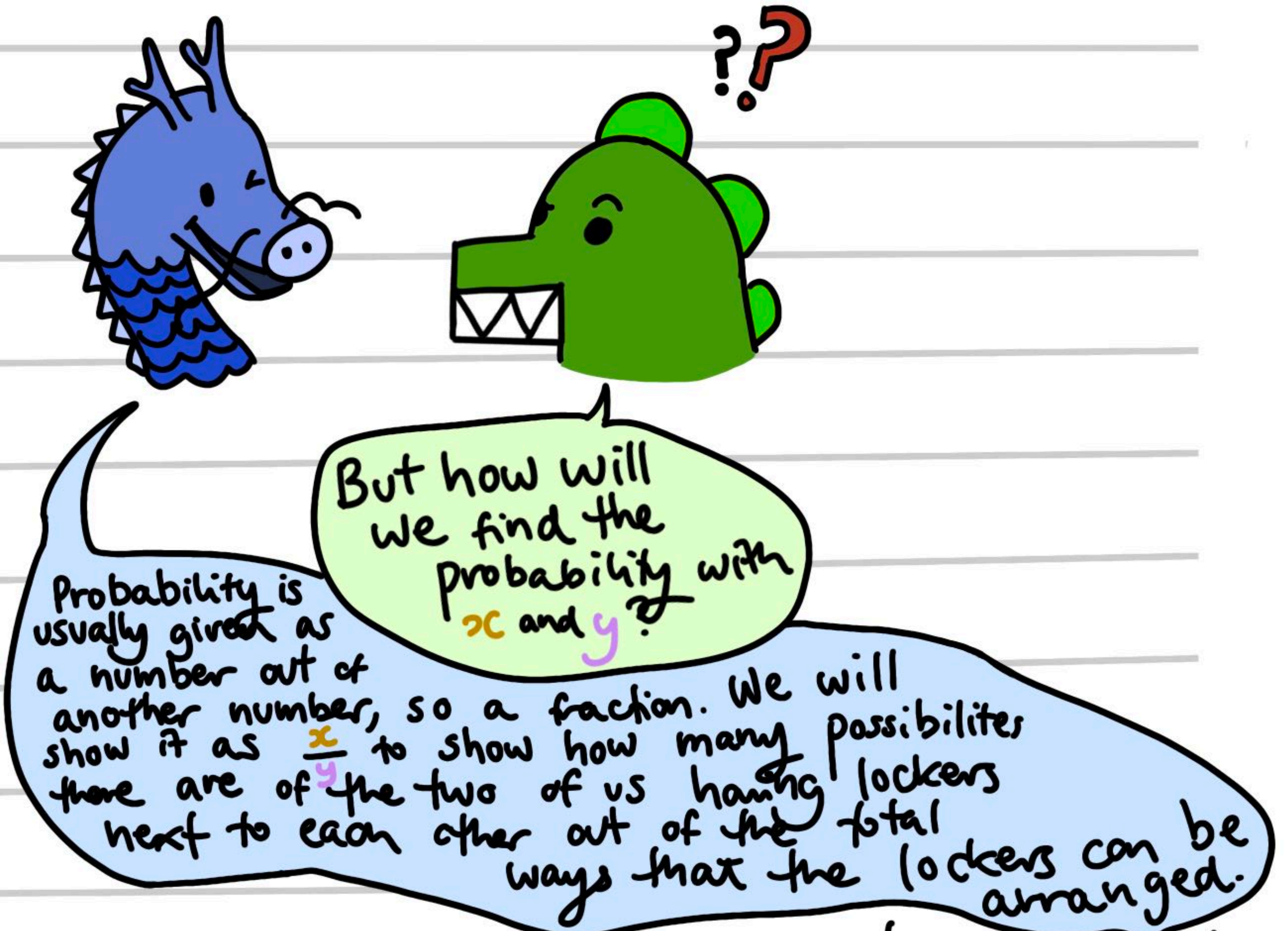
After all, they use their lockers often...
Billy has to hold his rucksak in his claws because of his wings!



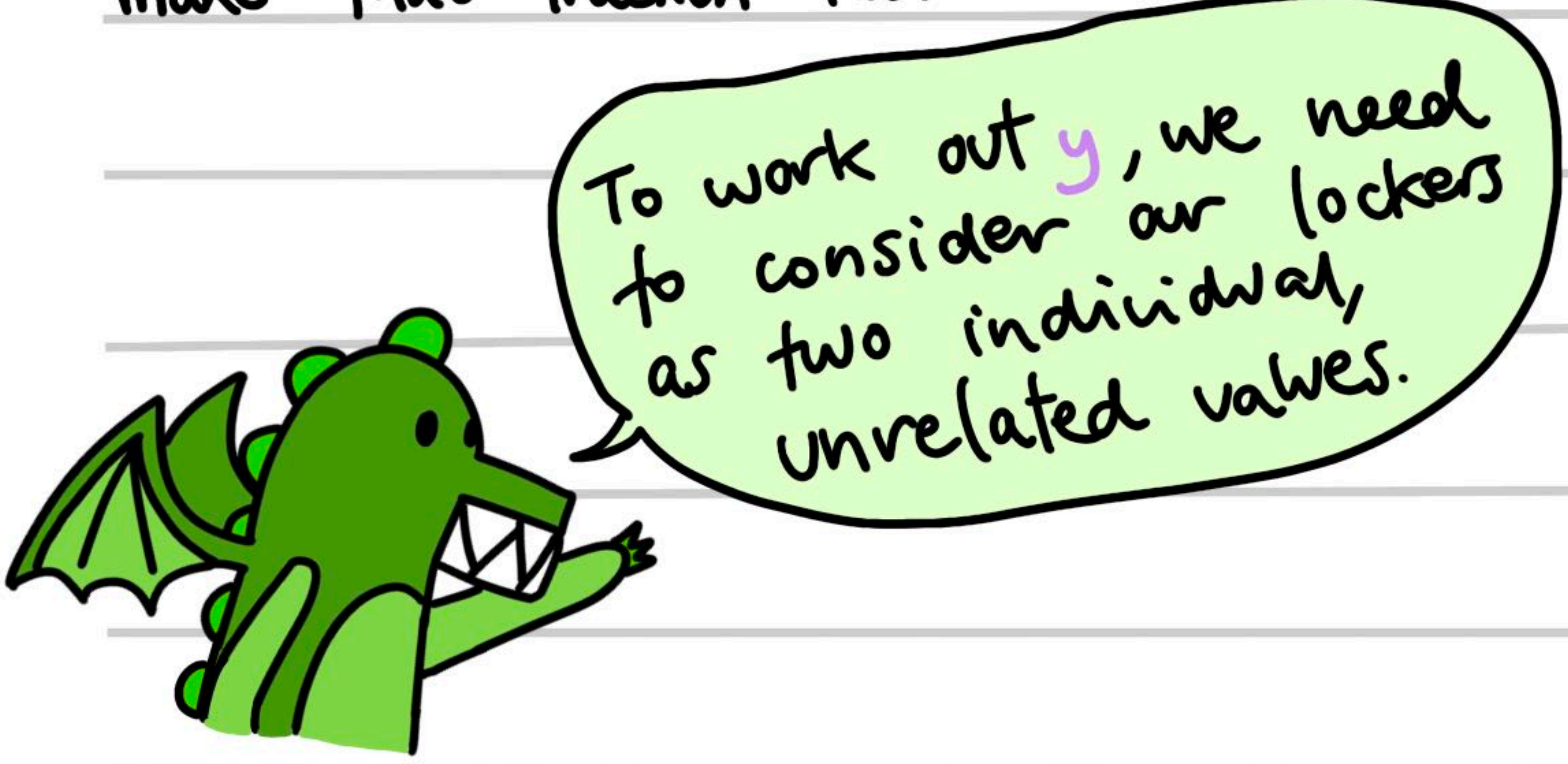
They knew that there were 840 lockers in the school, arranged in 140 sets of 6 lockers.

Representing their assigned lockers with a destined 'Billy' locker and a destined 'Loey' locker, they decided to find how many ways the 840 lockers could be arranged, y, and then work at how many ways the 840 lockers could be arranged if two lockers, Billy's and Loey's, were assigned next to each other, x.





Hold on, Loey! First, they needed to work out the values of x and y, so that they could make that fraction first.



How many ways can you arrange these three values?

A B C

Let's try to count them ourselves and see what we notice.

A B C
A C B
B C A
C B A
C A B
B A C

As you can see, there are 6 arrangements. $6 = 3!$, otherwise known as 3 factorial. $3!$ is $3 \times 2 \times 1$. So why is this the case?

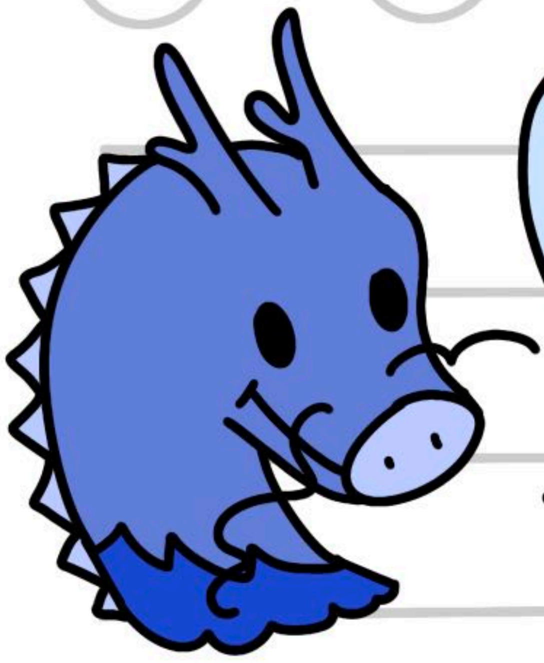
Let's look at it from this perspective. If there are three values, than every possible way for them to be arranged will be in three 'places', like this:

3 possibilities   2 possibilities   1 possibility

As you can see, the first 'place' could be any of the three values. The second 'place' could be any of the two values remaining, and the last 'place' would be the one value remaining. So, we multiply 3 with 2 with 1 to find the number of ways it can be arranged.

A	C	B
A, B, or C	B or C	B


So since we have 840 lockers, we have 840 values, so the number of ways they could be arranged is 840!, right?
Yup, which has 2094 digits! A very long number indeed. But rather than use 840! as our y value, we can use 6, because we're in the same class and will have the same set of lockers. And 6 factorial equals 720.
So if we need to have our two values, the 'Billy' locker and the 'Loey' locker, placed together then how do we work out the number of arrangements?



Let's refer back to our example of A, B and C.

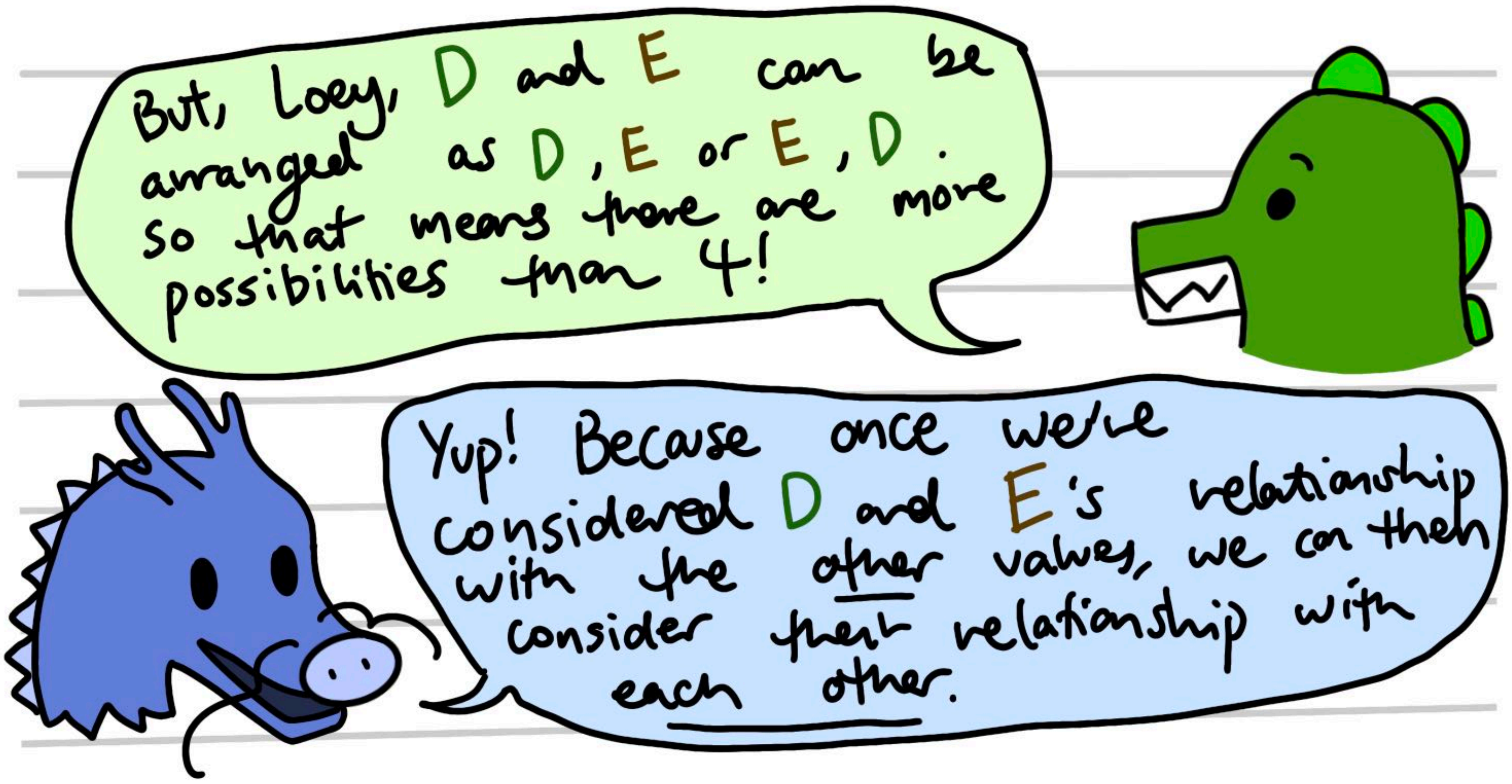
We'll add two values: D and E.

A B C D E

Let's say that the values D and E have to be next to each other. Then, we can consider D and E temporarily as one value.

A B C $\frac{D}{E}$

So, like this the number of ways the values can be arranged is 4! because we consider D and E as the same value.

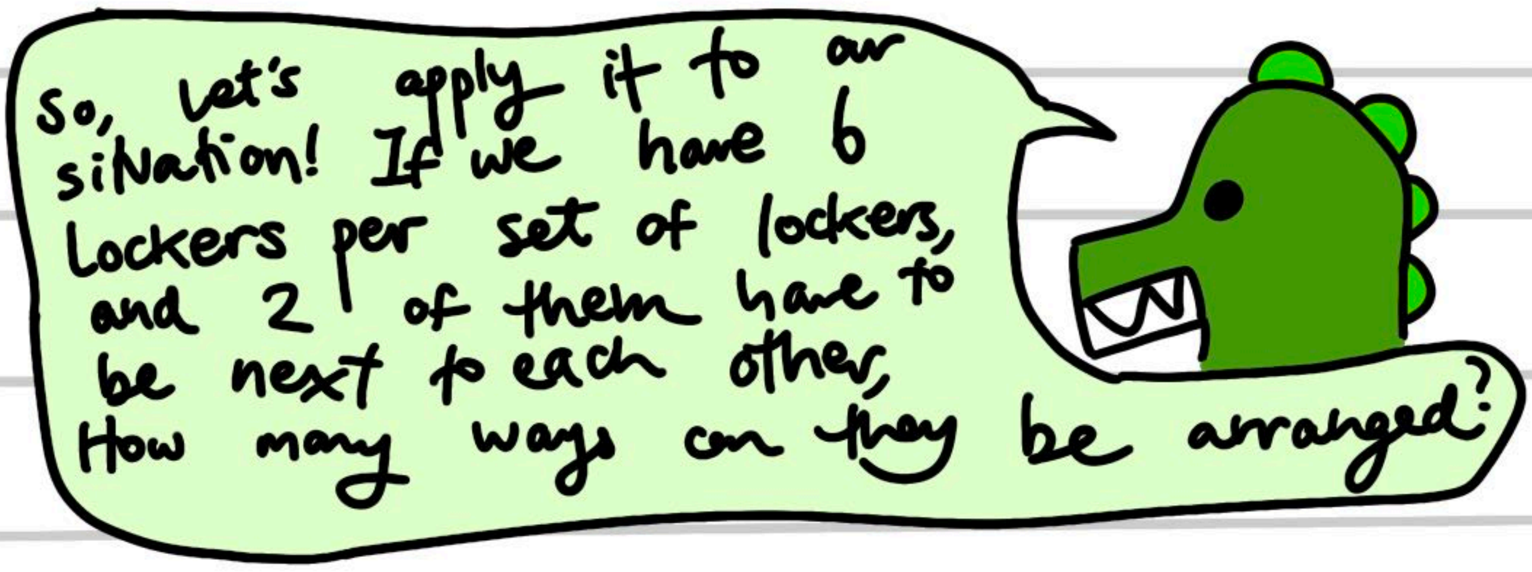


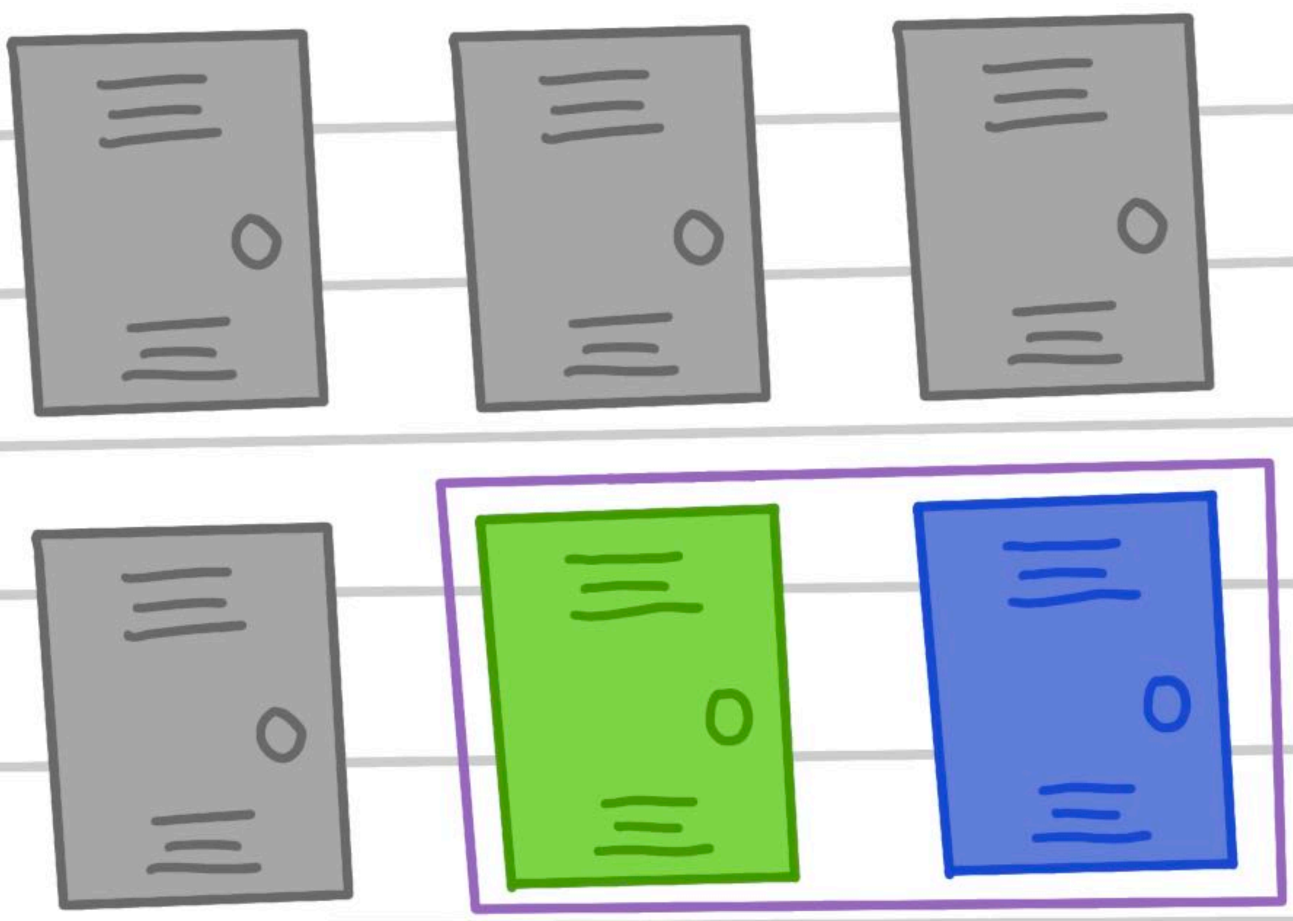
The number of arrangements for D and E are $2! = 2 \times 1 = 2$

D   E

D or E   E

And so, the number of ways A, B, C, D, and E can be arranged is $(4!) \times (2!)$ because the values can be considered as arranged in two separate 'sets'.

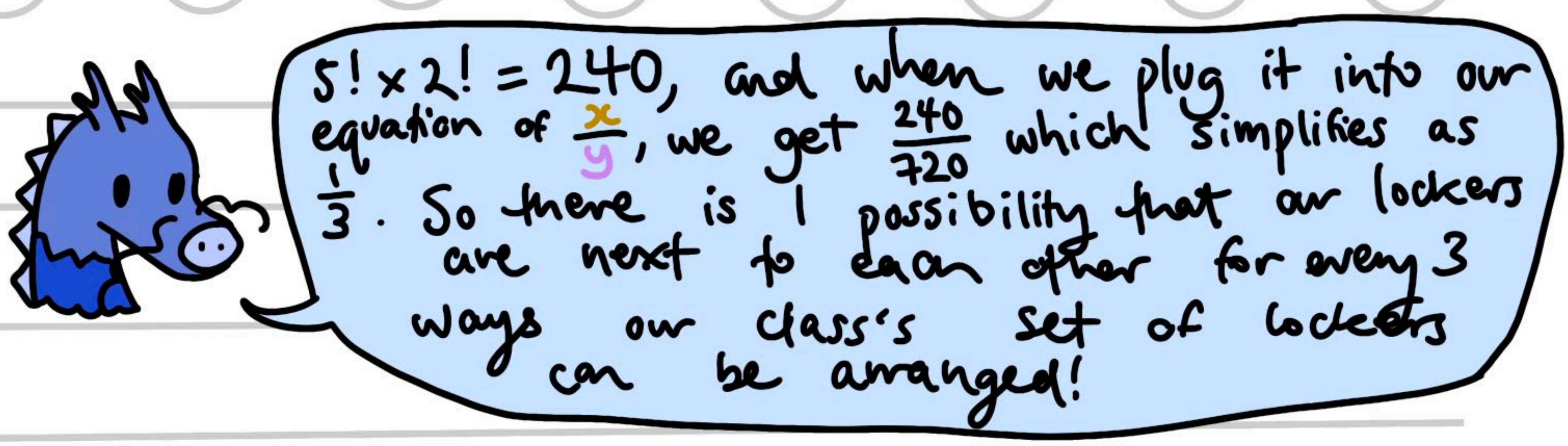




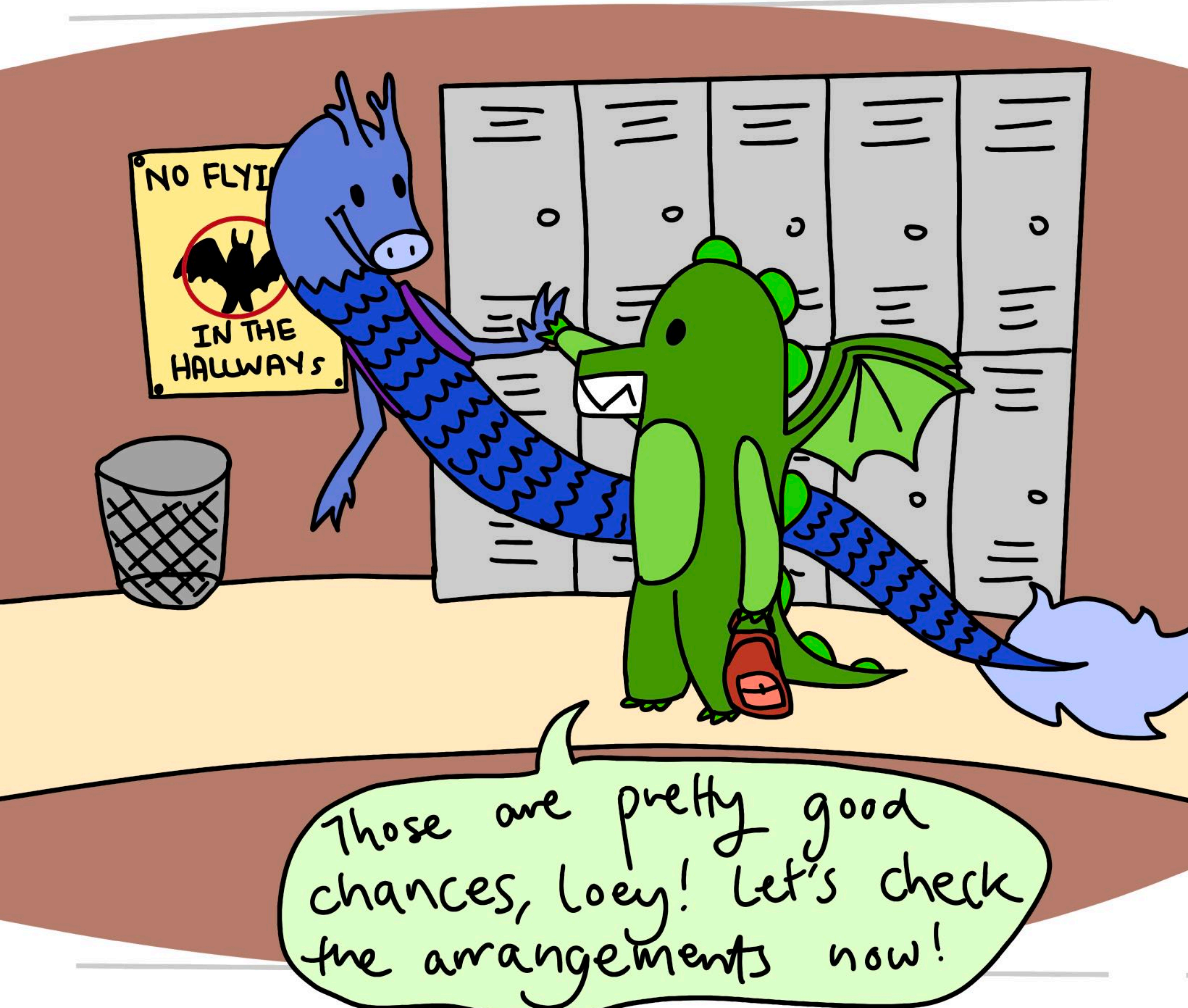
First we consider Billy's and Loey's lockers to be the same value, and result with 5!

We then consider them as a set, and result with 2!, so the answer is

$$\boxed{x = (5!) \times (2!)}$$



$5! \times 2! = 240$, and when we plug it into our equation of $\frac{x}{y}$, we get $\frac{240}{720}$ which simplifies as $\frac{1}{3}$. So there is 1 possibility that our lockers are next to each other for every 3 ways our class's set of lockers can be arranged!



NO FLYI
IN THE HALLWAYS
Those are pretty good chances, Loey! Let's check the arrangements now!

Billy and Loey are best friends. They want their lockers to be next to each other, but how will they know if this will be the case when they go back-to-school soon?

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