



**THE 2023 YOUNG MATHEMATICAL STORY AUTHOR (YMSA) COMPETITION**

**THE CINDY NEUSCHWANDER AWARD  
(THE 12-15 YEARS OLD CATEGORY)**

**LONGLISTED**

'The Hat Mystery' by Ella Zhao (12 years old)  
at Dulwich College Beijing (China)

You can read the author's inspiration for the story and the judges' comments  
on:

[www.mathsthroughstories.org/ymsa2023](http://www.mathsthroughstories.org/ymsa2023)

**#YMSAMaths**

# THE HAT MYSTERY

A SHORT MATHS STORY ABOUT POSSIBILITY  
AND REASONING

By: Ella Zhao DCB

After school, 3 best friends, Eliana, Erin, and Charlotte, were walking down the street looking for a hat shop. They needed hats for an event tomorrow. At the end of the street, they saw a hat shop with a giant sign sticking out that said:

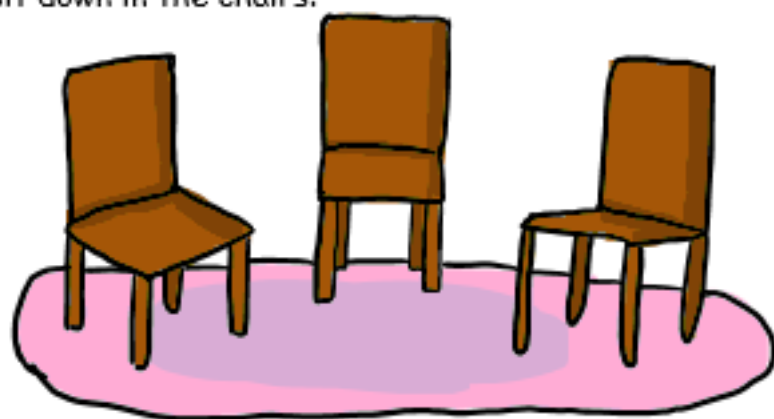


"This is perfect! Let's give it a shot, maybe we'll actually win!" Said Charlotte, excited.

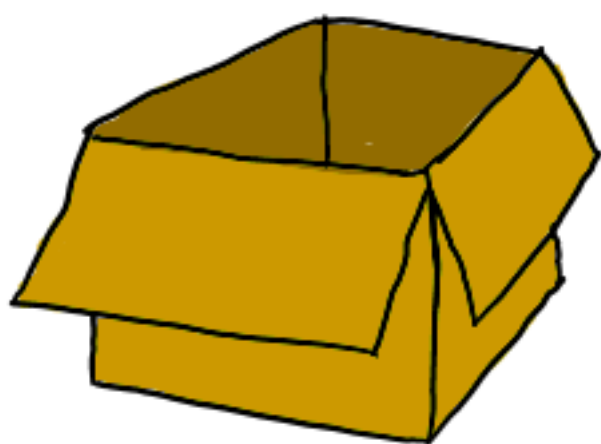
They pushed the door open and walked into the shop. As the friends walked up to the counter, Eliana asked the worker, "Hi, is it possible for us to attempt the colored hat challenge? The one on the sign."



The worker replied "Yes, of course!". She then motioned for the three kids to sit down in the chairs.



The worker then took out a box of hats from the corner and explained the rules, "There will be 5 hats of white or black. For each round, you will close your eyes and I will place a hat on your head. Then you will open your eyes and try to guess the color of your hat. Before the round starts, I will tell you how many white hats and black hats there are. You can see the hat color of the but not your own. There will be three rounds in total, and you must pass all three rounds in order to win the free hats."



They closed their eyes, and the worker placed the hats on their heads.



When they opened their eyes, the worker explained, "In this round, There will be 2 white hats and 3 black hats."



Eliana then looked around. Both of Erin and Charlotte's hats were white. 2 of the 5 hats were white, thought Eliana, and both are on Erin and Charlotte's heads, which means that my hat must be white.



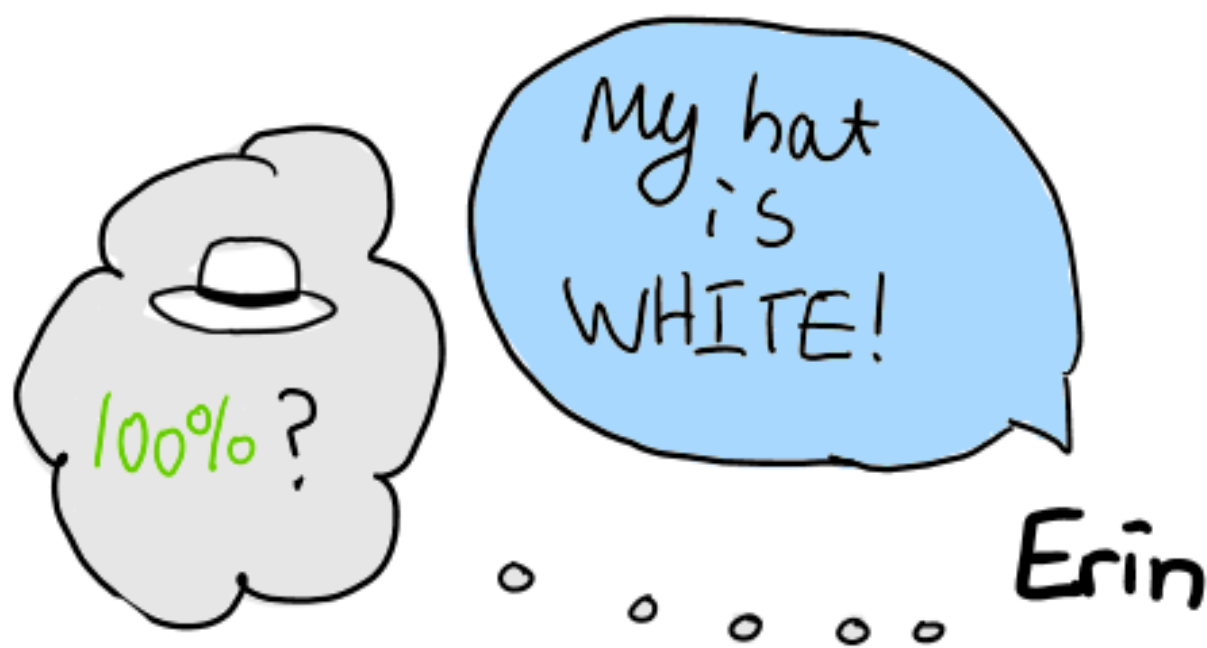
She then confidently replied "The color of my hat is black." Erin and Charlotte then quickly guessed the correct color of their hats.

The second round was way more difficult.

There were 2 black hats and 3 white hats, and Erin and Charlotte each wore a different color hat. Erin's was white and Charlotte's was black.



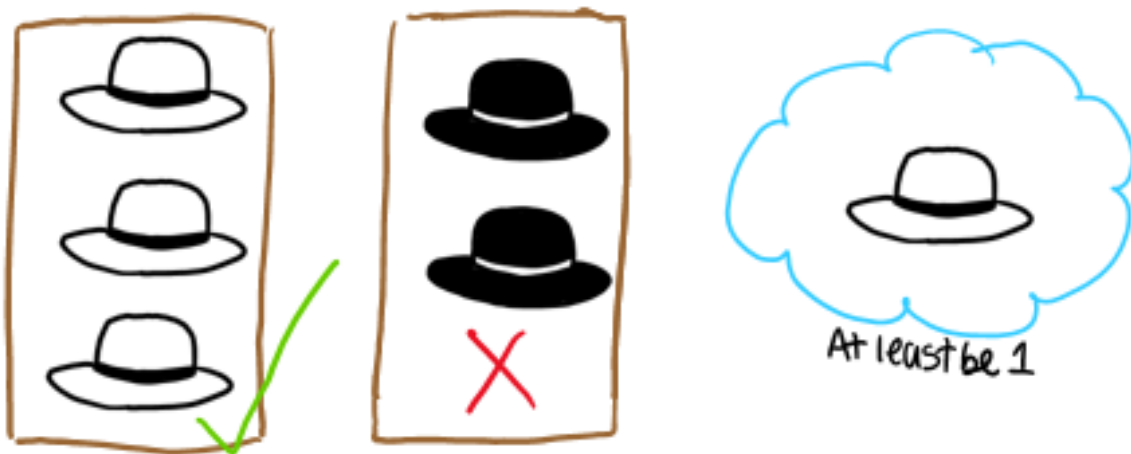
Eliana knew that there was 1 black hat left and 2 white hats left. She knew that there was a higher possibility of her getting a white hat, but that doesn't mean that she wouldn't get the black hat by any chance. Just as she was thinking, Erin called out, "The color of my hat is white." Eliana looked at the worker, and she nodded. Now she has a new piece of information, that Erin knew his hat was 100% white. But how?



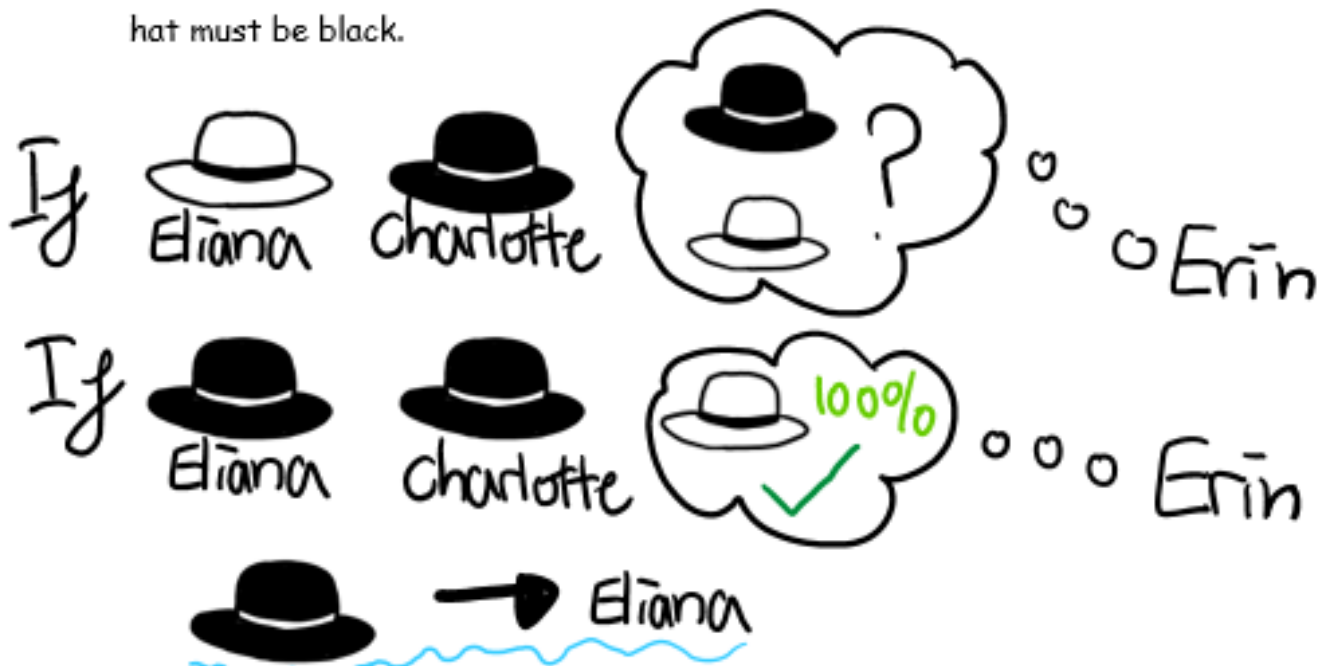
Eliana decided to picture a drawing in her head. There were 2 black hats and 3 white hats in total.



All three kids could be wearing a white hat but not everyone can wear a black hat at the same time, as there are only 2, which means that one hat must be white no matter what.



If her hat was black and Charlotte's hat was white, then Erin wouldn't have known if his hat was black or white. However, if both of their hats were black, then for sure Erin's hat would be white. Therefore, Eliana's hat must be black.



The second round wasn't that hard, after all.

For the final round, there were 2 white hats and 3 black hats.



Eliana opened her eyes and saw that both Erin and Charlotte's hats were black. That meant that her hat could either be black or white, and she wouldn't know what color her hat actually was. She would have to listen and wait for what the others were going to say and use that information to figure out what color her hat is, just like the previous round. However, when it was time to say their guesses out loud, both Erin and Charlotte replied, 'I don't know'.





Eliana sat back and thought. This meant that at least one of Charlotte and her own hat was black, or else Erin would have guessed easily that his hat was black.



Now it was Charlotte's turn to answer. Using the information Erin gave, Charlotte knows that one of their hats must be black, but she still replied 'I don't know'. If Eliana's hat was white, then for sure Charlotte's hat would be black, as that would fit perfectly with Erin's answer, but Charlotte still replied, 'I don't know'. This could only mean that Eliana's hat is also black.

If  → Eliana, then  100% → Charlotte

But,  which can only mean:  → Eliana

When it was her turn to guess, Eliana replied confidently, "The color of my hat is black." The worker nodded, then Erin and Charlotte quickly guessed the correct color of their hats as well, using Eliana's answer as a guide.



"Congratulations on passing all 3 levels!" Congratulated the worker, "As the reward for successfully completing the challenge, all 3 of you get to pick out any hat you like for free!"

"Yay! We did it!" Shouted the 3 children as they high fived each other. Then they headed off to pick their favorite hats.



Eliana picked a lavender hat with a bow in the middle,



Erin picked out a baseball hat with 'Maths' written on it,



And Charlotte picked out a spring themed flower hat.



The 3 friends thanked the worker again and walked out of the shop happily together, laughing and joking along the way.



Want to know how to solve the famous black and white hat riddle? Join Eliana, Erin, and Charlotte as they figure out the solution by using the exclusion method and their Maths knowledge of possibility and reasoning.

