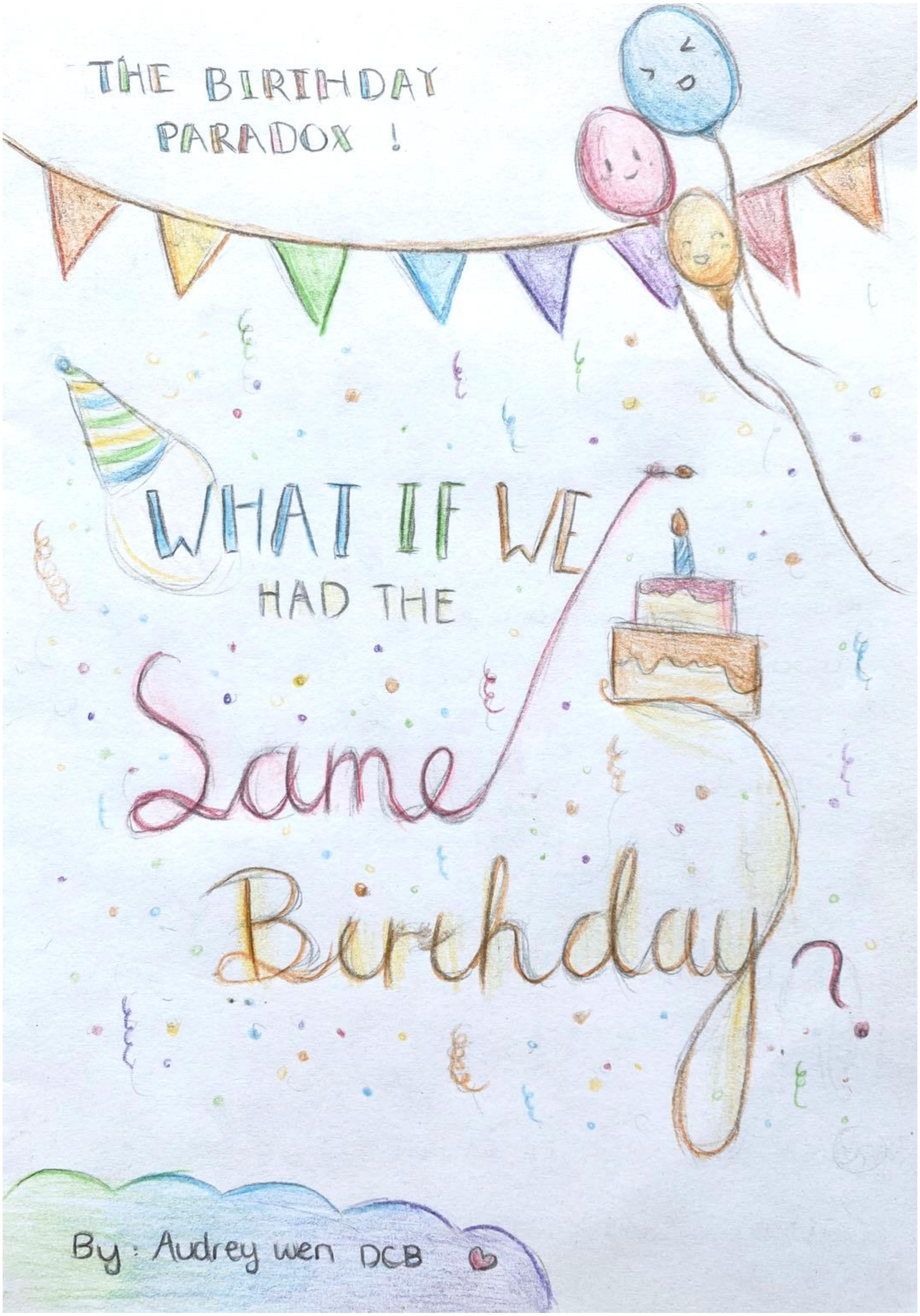

THE BIRTHDAY PARADOX !
WHAT IF WE HAD THE Same Birthday?
By : Audrey wen DCB

CHAPTER 1

The Birthday problem

Amy was so happy! It was her birthday today, she was turning 10!

Amy walked into her classroom and saw 2 birthday cakes on the table.

"Wait, why are there 2 cakes?" Amy asked, confused.

"Me and my friends can't really finish them all..."

The teacher smiled. "Don't worry. This one's for Robert! Today's his birthday too! Have you heard?" She replied.

"Robert?" Amy said, "I'll tell him happy Birthday later!"

"Sure!" The teacher said brightly, "Happy Birthday Amy!"

Amy walked out of the classroom with a big question mark in her head.

"How is this possible? Almost every group of people I meet would have at least 2 people sharing the same birthday. Last year in my camp group, in my Grade 3 class and even in my afterschool group math class! How can I meet many people with the same birthday so many times?" She thought to herself.

dot, dot, dot... And... so many times that Amy can't even remember...

After a long moment of trying to remember what exactly happened, Amy seemed to have found a pattern!

"Aha! Because every class that has people who shares a birthday with someone else has 23 people in total!" she exclaimed at last.

But then, her excitement dropped to a frown.

"But my Grade 2 class had 23 people and no one shared the same birthday!" Amy muttered.

Amy graoned. She hated it when she had a desprate question and she couldn't find an answer to it.

"Amy, it's your birthday today, don't let this get in your way, be happy!" She told herself, trying to shake off her anxiousness. "But I will find out the answer afterschool!" Amy declared. She was sure of it!

CHAPTER 2

I can do this!

Amy went home with a handful of birthday presents and a belly full of cake.

She put all of her presents on the carpet,

"A new painting set! My favourite board game! My friends are the best!" Said Amy, oohing and aahing at every present.

How eager she was to play with them!

"No! I have to find out the answer first!" She shouted.

Amy stopped herself and walked into her room, still trying to steal a few looks at her precious presents.

Amy sat down at her table and took out her notebook and her pen.

"In a group of 23 people, sometimes there would be people who shares a birthday and sometimes there wouldn't, which means there must be a probability of there being any!" Amy said excitedly, like a light bulb has suddenly poped on in her head.

Luckily, Amy still remember doing similar types of questions as this problem in her group math class.

"I can do this!" She said, and got to work...

CHAPTER 3

what is the probability?

Amy started writing things down on her notebook:

> Probability of people sharing a birthday + probability of people not sharing a birthday = 100%
>
> so...
>
> 100% - probability of people not sharing a birthday = Probability of people sharing a birthday

Ignoring twins & leap years

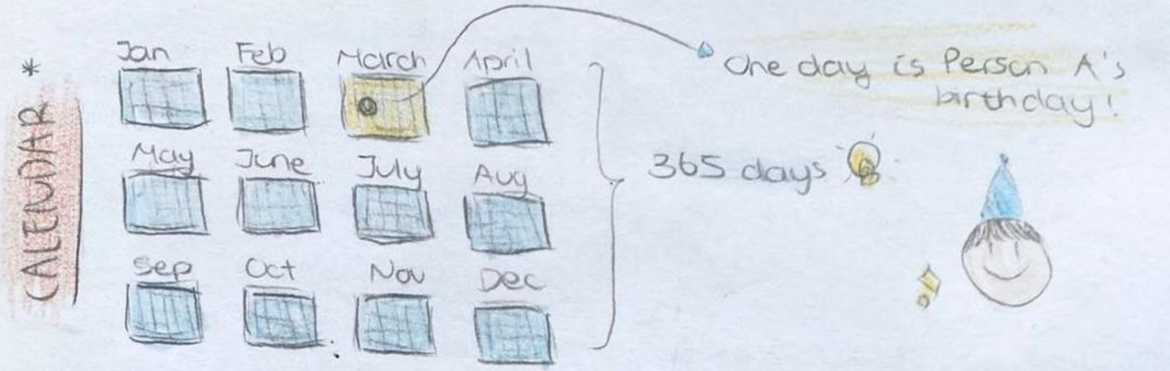
"Okay, I'll start by calculating the probability of no one sharing a birthday in a group of 23 people." Amy thought.

she closed her eyes and imagined two people, person A and person B, has different birthdays.

Person A   Person B

They have <u>different</u> birthdays!

"1 day of the year would be Person A's birthday." Amy said.



"This leaves only 364 days left to be Person B's birthday!" Amy continued, "This means that the probability of Person A and B not sharing the same Birthday is 364/365, which is... 99.7%" Amy typed on her calculator.

$$\frac{364}{365} = 0.997 = 99.7\%$$

"I'm doing it!" Amy said cheerfully. "Okay, now I will bring in person C, which will have a different birthday with person A and B."

"Now there's only 363 days left for person C's birthday. The probability of person A, B and C having different birthdays is now 363/365. It would be 362/365 for person D and so on! The probability gets lower and lower when there are more people!" Amy pointed out. "Now I will multiply all 23 people's 23 probabilitis to get the probability of no one sharing a birthday in a group of 23 people!"

Amy calculated the probability on her notebook:

$$\frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} \times \frac{362}{365} \times \frac{361}{365} \times \frac{360}{365} \times \frac{359}{365} \times \frac{358}{365} \times \frac{357}{365}$$
$$\times \frac{356}{365} \times \frac{355}{365} \times \frac{354}{365} \times \frac{353}{365} \times \frac{352}{365} \times \frac{351}{365} \times \frac{350}{365} \times \frac{349}{365} \times \frac{348}{365}$$
$$\times \frac{347}{365} \times \frac{346}{365} \times \frac{345}{365} \times \frac{344}{365} \times \frac{343}{365} = 0.4927 = 49.27\%$$

"There we have it! 49.27% is the probability of no one shari a birthday!" Amy said happily. That was alot of writing to do

"Now just substract it from 100%!" Amy said.

$$100\% - 49.27\% = 50.73\%$$

"Yay! I finally got it! The probability of some one sharing birthdays in a group of 23 people is 50.73%!" Amy squealed, jumping up and down.

But something was still bothering her. Another burning question to answer.

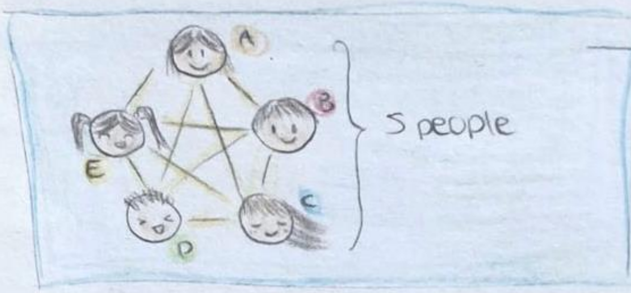
How can there be such a high probability in only 23 people ???

CHAPTER 4

Another question to answer

"Okay, I'm gonna work out how many comparisons 23 people can make all together. It's really hard to believe only 23 people could have over 50% chance of having someone sharing same birthdays!" Amy muttered.

"I'll start with only 5 people." Amy said.



NOTE

Every line between these 5 people is a comparison 2 people make, a chance that someone would share a birthday!

"Each of the 5 people can be paired with any other 4!" Amy thought very carefully, "But I can't just do 5×4, I have to divide it by 2! Half of 5×4 is repeated, because pairing A with B is the same as pairing B with A!"

pair A with: B, C, D, E → 4 people
Pair B with: ~~A~~, C, D, E → 3 people
pair C with: ~~A~~, ~~B~~, D, E → 2 people
Pair D with: ~~A~~, ~~B~~, ~~C~~, E → 1 person
Pair E with: ~~A~~, ~~B~~, ~~C~~, ~~D~~ → E has already been paired with everyone ⍨ 0 people ⍨

Amy counted how many people that are paired with someone and also how many people that are already paired - she was right! They both equal 10! Which means that she can do (5×4)÷2 to work out how many comparisons/pairs there are!

"I got the formula of calculating the number of comparisons! This would be really useful!" Amy said, smiling.

She wrote down her formula on her notebook...

Formula of finding the number of possible pairs a group of people can make:

$$\frac{\text{The number of people} \times \text{The number of people a person can pair with}}{2}$$

"Wahoo! I found the formula! Now, all I need to do is use it to figure out how many comparisons 23 people could make!" Amy said. She was excited and nervous at the same time. She's about to find out the answer!

$$\frac{23 \times 22}{2} = 253 \text{ comparisons}$$

!

Amy almost fell out of her chair when she saw the answer. "Wow, 253 comparisons? Now it totally make's sense! Every one of them is a chance of 2 people sharing the same birthday!" Amy exclaimed.

CHAPTER 5

Good job Amy!

Amy was so proud of herself!

"I finally understand now!" She cried happily, and walked out of her room. She didn't forget about her presents she really wanted to play with earlier!

She walked into the dining room to grab something to eat. The room was dark, but Amy could see some candle light...


H A P P Y B D A Y
HAPPY BIRTHDAY AMY!
Wow.

Amy was turning 10 today! ♥

When she finds out that she was not the only one with a birthday today in her class, she starts to wonder: How can she meet so many people with the same birthday? Let's go and find out how curious Amy discovers the answer!