Children as Producers of Mathematical Picture Books: A Teaching Approach to Consolidate Children's Conceptual Understanding of Multiplication

Natthapoj Vincent Trakulphadetkrai

Institute of Education, University of Reading, 4 Redlands Road, Reading RG1 5EX, UK

E-mail: n.trakulphadetkrai@reading.ac.uk

Tel: +44(0)1183782665

Abstract

This paper reports the findings of a short-term intervention study that sets out to explore the effectiveness of having Year 4 (8-9 years old) children in England produced their own multiplication-related picture book to consolidate their conceptual understanding of the topic. The Analysis of covariance (ANCOVA) analysis, while controlling for the pre-test scores, found children in the intervention class (N = 27) significantly outperformed their peers in the comparison class (N = 25) on the Procedural Fluency scale (F[1, 49] = .8.19, p = .006, η^2 = .14) and the Overall Representation scale (F[1, 49] = 13.54 p = .001, η^2 = .22), which – as the current study argues – is an indicator of children's conceptual understanding in mathematics. These preliminary findings are promising and practitioners may find teaching mathematics through having children produce their own mathematical picture book an effective pedagogical approach.

Keywords

Conceptual understanding; constructionism; external representation; multiplication; picture books

Introduction

The recently revised primary mathematics curriculum for England (Department for Education, 2013) has set a high expectation for what young children should be able to achieve. One such expectation is concerned with multiplication. Whereas previously children were only expected to "recall multiplication facts to 10 x 10" by the end of Year 6 (11 years old) (Department for Education and Employment, 1999, p. 69), the new expectation is that children should now master the 12 times table as early as the end of Year 4 (9 years old). Arguably, such expectation has put a great deal of pressure on primary teachers to deliver, which could result in an overreliance on rote memorization, at the expense of teaching for conceptual understanding. It is thus even more imperative than ever for these teachers to be supported with effective pedagogical strategies that can aid young children to develop not only procedural fluency, but also conceptual understanding of multiplication.

This study reports the finding of an innovative mathematics teaching and learning strategy whereby young children produce their own multiplication-related picture book. The assumption is that by encouraging young children to embed their knowledge of multiplication in a meaningful context and by getting them to represent multiplicative situations visually, these two key features of picture book creation will allow them to effectively develop their conceptual understanding of multiplication.

Literature Review

Conceptual understanding

Key to effective mathematics learning is the need for conceptual understanding. Kilpatrick, Swafford and Findell (2001) define *conceptual understanding* as the ability to represent mathematical situations in different ways, and the degree of students' conceptual understanding can thus be measured by examining "the richness and extent of the connections

[between representations] they have made" (p. 119). Similarly, Hiebert and Carpenter (1992) suggest that "mathematics is understood if its mental representation is part of a network of representations. The degree of understanding is determined by the number and strength of its connections" (p. 67). More recently, Barmby, Harries, Higgins and Suggate (2007) propose that mathematical understanding be understood as a network of representations associated with a mathematical concept (p. 42). In line with these definitions, conceptual understanding will be taken, in this study, to refer to an ability to make connections between different mathematical representations.

It is often useful when discussing conceptual understanding to do so by contrasting it with a closely related mathematical concept, namely *procedural fluency*, which can be defined as "knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently" (Kilpatrick et al., 2001, p. 121). To further distinguish these two constructs, Skemp (1991) used *instrumental understanding* and *relational understanding* to explain procedural fluency and conceptual understanding respectively. While the former is taken to refer to "rules without reasons", the latter is taken to refer to "knowing both what to do and why" (p. 2). Following Dehaene (1997), Ramos-Christian, Schleser and Varn (2008) argue that learners who struggle with fluency have less cognitive resources (e.g. attention and working memory) to aid their comprehension. Subsequently, this highlights the complementary relationship between these two constructs.

Mathematical representations

Mathematical representation is key to develop mathematical abstraction, which according to Dienes (1973), can help to unlock children's mathematical development. Representations can manifest in two ways: *internally* and *externally*. While internal representations are "abstractions of mathematical ideas or cognitive schemata that are developed by a learner through experience" (Pape & Tchoshanov, 2001, p. 119), external

representations are concerned with mathematical representations that can "act as stimuli on the senses" (Janvier, Girardon & Morand, 1993, p. 81). Simply put, while internal representations are private to the learner, external representations can be *externalised* and shared with or experienced by others. In the context of this study, the focus will be on exploring mathematics learners' external representations as they can be examined externally. One of the key benefits of external representations is computational offloading. By reducing the demand on students' cognitive effort or working memory, they are able to focus more on understanding and solving the problems.

There is a range of external representations available for mathematics learners to use. Bruner (1966), for example, argued that students can represent their (mathematical) thinking in three different ways: *enactively* (e.g. using manipulatives or concrete objects); *iconically* (e.g. representing mathematical concepts or processes visually); and *symbolically* (e.g. through the use of mathematical notations). Building on Bruner's (1966) modes of representations, Haylock proposes a slightly extended framework (Haylock, 1982; Haylock, 1984; Haylock & Cockburn 2013), connecting concrete materials, pictures, and symbols with a fourth element, *language*, which encompasses, for example, reading instructions or using specific mathematical words. To an extent, the usefulness of these frameworks to help develop and assess children's mathematical understanding is arguably limited by its lack of emphasis on the role of meaningful context in which learning needs to be embedded. Yoong (1999) refers to this missing component as the 'story' component in his multi-model strategy framework, which itself is an extension of Haylock's (1984) original framework. By getting students to create a story, such as word problems, to accompany a number sentence, Yoong (1999) argues that it can bridge the gap between textbook mathematics and real-world applications.

In the context of this study, the connections between three of these key mathematical external representations, namely *symbolic representation (e.g. number sentences), visual*

representation (e.g. mathematical diagrams), and *contextual representation (e.g. word problems)* will be further explored in the following sections. While other types of representation, such as concrete representation through manipulatives, are also important, a decision was made to focus on the three aforementioned representations. This is primarily due to the labour intensive nature of collecting data involving concrete representation, and the practical constraints of the current study (particularly in relation to human resources and funding).

Multiplication and its symbolic, visual and contextual representations

The learning of multiplication can be a daunting task for young children, particularly when multiplication can be found in several *situations* and has different *properties*. Concerning the former, leading figures in this field, such as Greer (1992), proposed four key situations that involve multiplication of whole numbers and that can also be represented visually in different ways, namely: *equivalent groups* (e.g. 3 tables, each with 4 children); *rectangular arrays* (e.g. 3 rows of 4 children); *multiplicative comparison* (e.g. 3 times as many boys as girls) and *Cartesian product* (e.g. the number of the different possibilities for girl-boy pairs from 3 girls and 4 boys). Closely related to these visual representations of multiplicative situations is the *contextual* representation of the operation. Haylock and Cockburn (2013) highlight that children often struggle to embed multiplication in an everyday context due to their lack of understanding that the numbers must normally represent different sorts of things. For example, when representing 6 x 4 contextually, if a child has chosen 6 to represent six children, then 4 has to represent something other than children. To an extent, this might be due to their overgeneralization of the addition and subtraction concepts where the different numbers can represent the same sort of thing e.g. two books plus three books. From their analysis of hundreds of contextual representation of multiplication written by children aged 9 to 11 years old, the result shows that "only a small proportion of them seem to have clear structures in their

mind that they can connect with multiplication. Children seem to lack any kind of picture of what is going on when two numbers are multiplied together" (Haylock & Cockburn, 2013, p. 98).

In terms of *properties*, multiplication has three distinctive properties: *commutative, associative and distributive*. In the context of primary school-aged children, the commutative property is arguably the most relevant, but also problematic. For example, several mathematics teachers, mathematics teacher educators and mathematics education textbooks still maintain that when multiplication number sentences are represented contextually or visually, the order of the numbers becomes crucial. While some argue that the *multiplicand* (the first number) and the *multiplier* (the second number) should be taken to refer to the number of sets and the number of objects in that set respectively, others argue the opposite (Lamb, 2015). Thus, for example, while some argue that 6 x 4 should be taken to mean 6 sets of 4 objects, others argue that this be interpreted as 4 sets of 6 objects. In the context of this study, either of these interpretations and hence representations are valid.

With these different multiplicative situations and properties in mind, it soon becomes apparent why many young children may find it challenging to grasp the concept of multiplication conceptually, particularly when attempting to represent this arithmetic operation in different ways. To the best of the researcher's knowledge, previous empirical research on learners-generated external representations of multiplication is surprisingly limited. For example, one particular group of researchers in the UK (Barmby, Harries, Higgins & Suggate, 2009; Harries & Barmby, 2006; Harries & Barmby, 2007) extensively explored (Years 4 and 6) children's use of external representation to solve multiplication problems. These studies asked children to use a piece of software to illustrate how they would use the array method to solve multiplication problems. However, the relevance of these studies to the current study is

somewhat limited due to the fact that children were asked to use a particular type of visual representation.

Mathematics learning through picture books

Due the complexities outlined previously, some children can find learning about multiplication challenging. This calls for an innovative teaching and learning approach that aids the development of children's conceptual understanding of the topic by getting them to make meaningful connections between the different external representations of multiplication. This study argues that getting children to produce their own picture book with a narrative relating to multiplication can help develop their conceptual understanding of the topic.

The integration of stories (particularly in the picture book format) in mathematics instruction has traditionally been used to situate mathematics learning in a meaningful context (Billings & Beckmann, 2005). The act of embedding mathematics learning in a meaningful and familiar context allows children to see that mathematics is, in fact, a part of their everyday life experience and is not just a collection of abstract formulae and theorems.

In addition to the contextual feature of stories found in these picture books, visualisation through page illustration is also a key component. Lane (1980, as cited in Waugh, Neaum & Waugh, 2013) highlights three different ways in which pictures can interact with the narrative: 1) *graphic decoration* where illustrations simply beautify the text, but add very little to the content or meaning of the text; 2) *narrative illustration* where the illustrations closely match the narrative; and 3) *interpretative illustration* where the illustrations can be used to expand and enrich the narrative. Beyond these three different types of interaction, it is important to note that wordless picture books do also exist where the illustration *is* the narrative (Waugh et al., 2013). In the context of mathematics education, narrative and interpretative illustrations can be particularly useful to support mathematics learning and teaching as the context in which mathematical contents or processes are embedded can be represented visually and brought to

life. Additionally, as picture books are often beautifully illustrated, they can arguably help engage reluctant readers and assist those children who require support in decoding the text. Examples of picture books that can be used to aid children's understanding of multiplication include *'2 x 2 = Boo!'* (Loreen, 1995) and *'Multiplying Menace: The Revenge of Rumpelstiltskin'* (Calvert, 2006).

The effectiveness of using picture books to aid the mathematics learning process has rarely been explored empirically. Of those few studies (e.g. Elia, van den Heuvel-Panhuizen & Georgiou, 2010; Hong, 1996; Jennings, Jennings, Richey & Dixon-Krauss, 1992; van den Heuvel-Panhuizen, Elia & Robitzsch 2016; Young-Loveridge, 2004), none of these studies set out to explicitly explore how effective the use of picture books can aid children in making connections between different mathematical representations. Additionally, these studies were all conducted with very young children (5-6 years old) and the role of children in these studies is often limited to them being the *consumer* of stories, and are not given opportunities to apply their mathematical knowledge and understanding to produce their own mathematical narrative. This study proposes an intervention design in which older primary school children (8-9 years old) take on the role of the producer of stories, specifically in the form of picture books that are meaningful and interesting to them.

Theoretical framework

The very act of getting children to produce a mathematics story picture book is theoretically situated in Papert's (1993) theory of *constructionism*. Unlike constructivism, *constructionism* emphasises not only the process of internationalization, but also on *externalization*. Constructionists argue that construction of knowledge takes place both *in the head* (internalization) and supported by "construction of a more public sort 'in the world'" (externalization*),* whereby learners creating *a public artefact* of what they know that can be "shown, discussed, examined, probed, and admired" (Papert, 1991, p. 142). In turn, this process

helps to shape and sharpen the knowledge (Ackermann, 2010). In the context of the current study, such public artefact is a learners-generated multiplication picture book where knowledge and understanding of multiplication is embedded in the narrative.

More specifically, by allowing children to become the producer of these mathematics picture books, they are encouraged to understand in which meaningful context a given mathematical concept or skill can be applied, and how to visually represent them through their own mathematical page illustrations. Thus, not only would this experience afford teachers an opportunity to formatively assess their children's mathematical understanding and to make the cross-curricular links between mathematics and literacy, the current study would also argue that this mathematics teaching and learning strategy has the potential to enhance children's ability to make meaningful connections between the different modes of mathematical representations. The current study would also argue that this would ultimately lead to the development of children's conceptual understanding of mathematical concepts.

Current Study

Drawing from the aforementioned research gaps, the current study will thus address the following key research question: *To what extent does having Year 4 children produced their own multiplication-related picture book increase their conceptual understanding of the topic?*

Method

Procedure

This exploratory study is a small-scale intervention study, where two classes of Year 4 (8-9 years old) children were randomly assigned to either the intervention cohort or the comparison cohort. The data collection took place in June and July 2015. While the overall intervention period lasted for two weeks, the data relevant to this paper was collected across

five mathematics lessons in one week. This was preceded and followed by the pre- and post-test respectively.

The intervention, jointly planned by both the researcher and the intervention class teacher, involved the teacher beginning the first mathematics lesson by reading two multiplication-related picture books: *'2 x 2 = Boo!'* (Loreen, 1995) and *'Multiplying Menace: The Revenge of Rumpelstiltskin'* (Calvert, 2006) to the class. The children were then split into pairs and each pair was asked to create their own multiplication picture book, incorporating their knowledge of the operation as part of their storyline. This picture book was only meant to be 4-5 pages long to fit in with the available timeframe. On each page, the children were encouraged to use a combination of written words (contextual representation), page illustrations (visual representation) and number sentences (symbolic representation) (see Fig. 1 for an example). That said, the teacher did not interfere with children's creative process in choosing their own storyline, setting, and characters.

In relation to the comparison cohort, the teaching duration was identical to that of the intervention class i.e. 5 lessons across one week. The teacher in this cohort taught the way the lessons would normally have been taught, namely through primarily getting children to solve word problems on worksheets and class discussions. According to the teacher's lesson plan, initial teaching input for each lesson primarily involved demonstrating how to multiply 2- and 3- digit numbers with 1-digit number using the place value grid.
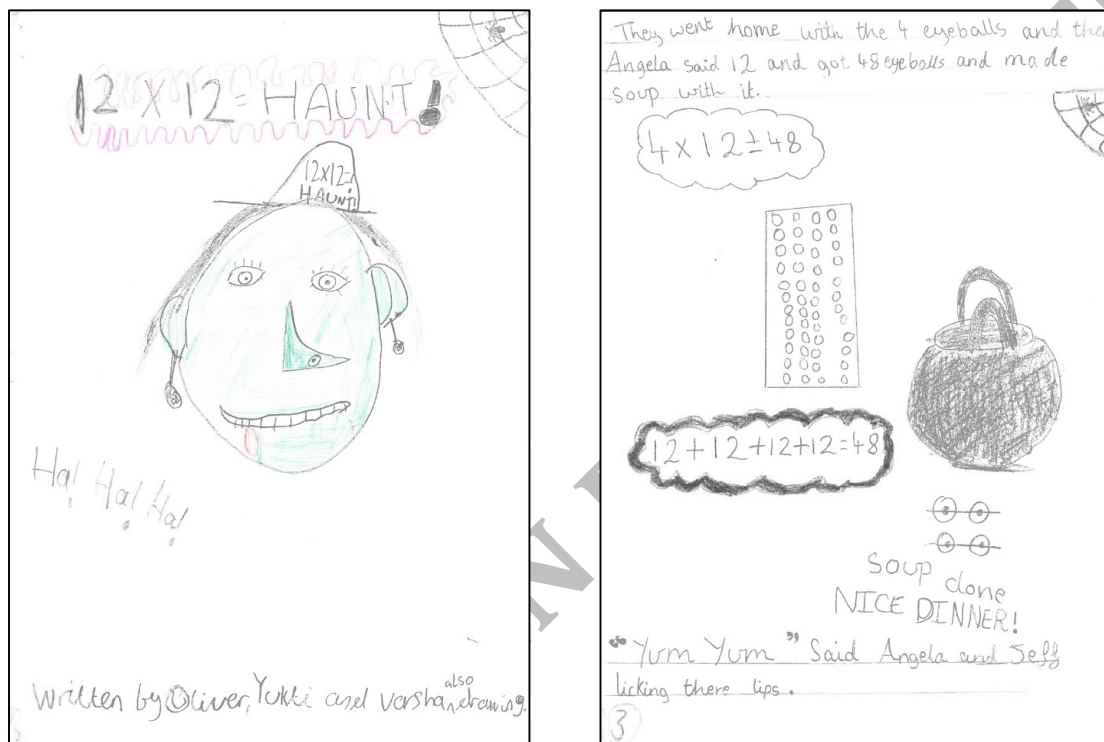

Fig. 1 Examples of picture books created by children in the intervention class

Participants

The data was collected from a local primary school, which is located in a suburban area in South East England. Around 60% of children come from a White British background, with only 6.9% of them being eligible for Free School Meals in 2014, comparing to the national 2014 average of 26.6% (Ofsted, 2014).

Sixty Year 4 (8-9 years old) children from two different classes agreed to take part and completed the pre-test, but 8 children, for various reasons, were not available to complete the

post-test. Of the remaining 52 children, 27 were in the intervention class (12 boys and 15 girls) and 25 were in the comparison class (13 boys and 12 girls). These children represent a range of reading, writing and mathematics ability levels, as assessed by the school.

Ethical clearance was granted by the Ethics Committee of the University of Reading's Institute of Education. Headteacher, the two Year 4 teachers as well as the children and their parents in this study were given an Information Sheet containing key information of the project and consent forms to sign (opt-out forms for parents). The identity of the children are protected and pseudonyms are used when reporting the findings.
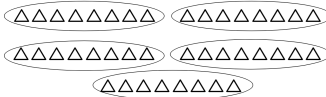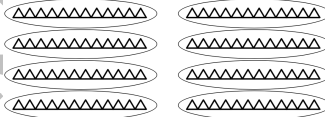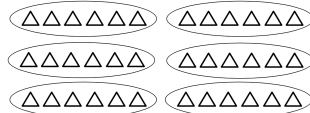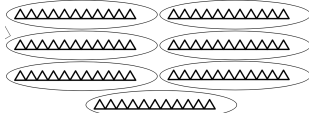
Measures

The content of the pre- and post-tests were developed by the researcher and were drawn primarily from England's national mathematics curriculum (DfE, 2013). As the data collection took place towards the end of the academic year, most of the Year 4 mathematics curriculum would have been covered. According to the Department for Education (2013, p. 25), the majority of Year 4 children should, by this stage, be able to: "*recall multiplication facts for multiplication tables up to 12 × 12; use place value, known and derived facts to multiply mentally; [...]; multiply two-digit and three-digit numbers by a one-digit number using formal written layout; solve problems involving multiplying and adding [...]*".

While the two tests were similar in their structure and challenging level, they were not identical. Each test contained 12 test items: only 6 items (i.e. Items 1, 3, 5, 7, 9 and 11) are relevant to the current study. Items 1 and 7 (on both tests) present children with a number sentence (e.g. 4 x 7) for them to represent it visually (e.g. diagrams) and contextually (e.g. word problems). Items 3 and 9 present the children with a word problem for them to represent it visually and symbolically (e.g. number sentences). An example of the number sentences given is *'Jane could jump seven times each minute. How many times could she jump in eight minutes?'*. Items 5 and 11 present the children with a visual representation (e.g. a diagram of

6 groups of 8 triangles) for them to represent it symbolically and contextually. The summary

of the test items can be found in Table 1.

Table 1
Summary of the pre- and post-test items

	Pre-Test	Post-Test
Item 1 *(a number sentence is given)*	4 x 7 = ___	3 x 8 = ___
Item 3 *(a word problem is given)*	Ten children are playing tennis together. Each of them brings four balls. How many balls do they have in total?	A town has nine schools. In each school, there are six teachers. How many teachers do the schools have in total?
Item 5 *(a diagram is given)*	5 groups of 8 triangles	6 groups of 6 triangles
Item 7 *(a number sentence is given)*	12 x 6 = ___	15 x 8 = ___
Item 9 *(a word problem is given)*	Beth exercises by running around the school eight times a day. How many times will she be running around the school if she ran every day for nine days?	Jane could jump seven times each minute. How many times could she jump in eight minutes?
Item 11 *(a diagram is given)*	8 groups of 12 triangles	7 groups of 11 triangles

Scoring procedure. To measure children's procedural fluency, a point is awarded if a child was able to work out a correct answer for each test item. As there are six test items, this resulted in the maximum score of 6 points. These items form the *Procedural Fluency (PF)* scale. It is important to note that a high score on this scale does not necessarily imply a high degree of conceptual understanding. This is because while some children are able to solve the arithmetic problems accurately, they may not be able to make appropriate connections between the different representations. In such example, the score would simply reflect their procedural fluency.

To measure children's conceptual understanding, three additional scales were used, namely *the Contextual Representation scale (CR), the Visual Representation scale (VR),* and *the Symbolic Representation scale (SR).*

To calculate the CR scale, a point is awarded if a child can represent each of the two number sentences (in Items 1 and 7) and each of the two diagrams (in Items 5 and 11) as *word problems* appropriately, resulting in a maximum of 4 points.

Similarly, to calculate the VR scale, a point is awarded if a child can represent each of the two number sentences (in Items 1 and 7) and each of the two word problems (in Items 3 and 9) as *diagrams* appropriately, resulting in a maximum of 4 points.

To calculate the SR scale, a point is awarded if a child can represent each of the two word problems (in Items 3 and 9) and each of the two diagrams (in Items 5 and 11) as *number sentences* appropriately, resulting in a maximum of 4 points.

The *Overall Representation scale (OR)* is made up of the scores of the CR, VR and SR scales, resulting in a maximum of 12 points (see Table 2 for the summary). This study would argue that the bigger the OR score, the more conceptual understanding is evident. The decision to design the OR scale as encompassing the VR, CR and SR scales was grounded in the literature which argue that conceptual understanding can be measured through the richness and

extent of the connections made between different representations of mathematical ideas
(Barmby et al., 2007; Hiebert & Carpenter, 1992; Kilpatrick et al., 2001).

Table 2
Summary of the Visual, Contextual and Symbolic Representation scales

Pre-Test	Post-Test	Representations	Contributing to …
Item 1	Item 1	Symbolic → Visual	the Visual Representation (VR) scale *(#1)*
		Symbolic → Contextual	the Contextual Representation (CR) scale *(#1)*
Item 3	Item 3	Contextual → Symbolic	the Symbolic Representation (SR) scale *(#1)*
		Contextual → Visual	the Visual Representation (VR) scale *(#2)*
Item 5	Item 5	Visual → Symbolic	the Symbolic Representation (SR) scale *(#2)*
		Visual → Contextual	the Contextual Representation (CR) scale *(#2)*
Item 7	Item 7	Symbolic → Visual	the Visual Representation (VR) scale *(#3)*
		Symbolic → Contextual	the Contextual Representation (CR) scale *(#3)*
Item 9	Item 9	Contextual → Symbolic	the Symbolic Representation (SR) scale *(#3)*
		Contextual → Visual	the Visual Representation (VR) scale *(#4)*
Item 11	Item 11	Visual → Symbolic	the Symbolic Representation (SR) scale *(#4)*
		Visual → Contextual	the Contextual Representation (CR) scale *(#4)*

An example of how the scoring works can be illustrated through Samantha's response
in Fig. 2. (Samantha is a pseudonym) The test item in question is Item 1 (pre-test) where
children were asked to represent 4 x 7 using a diagram and a word problem. Samantha was
able to correctly work out that 4 x 7 equals 28, so 1 point went towards her Procedural Fluency
(PF) scale. As she was able to represent 4 x 7 using a diagram appropriately (an array of 4 x
7), 1 point went towards her Visual Representation (VR) scale. However, she represented the
4 x 7 number sentence with the following word problem "A girl goes to the park, she finds

these coins 10p, 5p, 5p. She goes to the ice cream van she wants one, one ice cream costs 20p, does she have 20p? Yes." As Samantha's word problem did not quite represent the given number sentence, no point went towards her Contextual Representation (CR) scale.

PUZZLE 1

1.1) Number Sentence

4 X 7 = _____

Show how you work out the answer in the boxes below:

Informal jotting:

7 + 7 + 7 + 7 = 28

Formal written method:

7
×4
28

1.2) Image
Can you represent the number sentence above *visually?*

1.3) Word Problem
Using the number sentence above, can you create your own word problem?

A girl goes to the park, She finds theese coins 10p, 5p, 5p, She goes to the ice cream van she wants one, one ice cream costs 20p, does she have 20p? yes

Fig. 2 Example of Samantha's response

Moderation. Moderation of the marking and hence scoring took place at three national and international conferences where the audiences were asked to go through some of children's responses and to decide whether they agree with the researcher's marking. Any disagreements were discussed and the resulting moderation informed the researcher's marking of the rest of children's responses. The three conferences were the British Society for Research into Learning Mathematics (BSRLM) conference in Reading, UK in November 2015, the International Congress on Mathematical Education (ICME) conference in Hamburg, Germany in July 2016 and the European Conference on Educational Research (ECER) conference in

Dublin, Ireland in August 2016. Altogether, around 40 mathematics and mathematics education academics in the audiences from the UK and several European countries were part of this moderation process.

From these moderation exercises, it soon became apparent that scoring children's word problems (contextual representation) was the most problematic of the three types of representations. More specifically, four types of children's word problems were highlighted and discussed to ascertain whether points ought to be awarded, and they can be described broadly as:

- 'Extension' – an example is when a child tried to represent the 3 x 8 number sentence as a word problem, they posed the following question: *"There are 20 people in a triangular room. There are 8 monsters guarding each three doors. The leader of the people wants to know if they are outnumbered. Are they outnumbered?"*;

- 'Division' – an example is when a child tried to represent the 15 x 8 number sentence as a word problem, they posed the following question: "You have 120 grapes. You split them into 15 groups. How many groups [grapes] are in each group?");

- 'Lack of *each*' – an example is when a child tried to represent the 3 x 8 number sentence as a word problem, they posed the following question: *"There are 8 jars of sweets with 3 sweets in them. How many sweets are there altogether?"*;

- 'Little to no context' – an example is when a child tried to represent the diagram of 6 groups of 6 triangles, they posed the following question: *"Tom needed to times 6 x 6. What is it?"*.

The majority of the moderators agreed that while word problems that fall into the first two types ought to be scored as 'appropriate', those that fall into the last two types should not be given any point. This is primarily because while multiplicative thinking is evident, even

implicitly, in the first two examples, the same cannot be said about the last two categories. The current study does not claim that such decisions are absolute and universal truths. They merely represent the study's attempt to be consistent and transparent in its marking and scoring procedures.

Data Analysis

   Scale reliability. A scale reliability analysis was performed to assess the reliability of the pre-test and post-test scales. The Cronbach's Alpha values (see Table 3) showed that the majority of the scales (both pre- and post-tests) were found to be moderately reliable. The Cronbach's Alpha value of the post-test Overall Representation scale in particular was found to be very high ($\alpha = .827$). It is not currently clear what might have caused the poor Cronbach's Alpha value ($\alpha = .231$) for the pre-test Visual Representation scale, so caution must be exercised when interpreting any results relating to that scale.

Table 3
The summary table of the scales' Cronbach's Alpha

	Overall Representation (OR) scale	Contextual Representation (CR) scale	Visual Representation (VR) scale	Symbolic Representation (SR) scale	Procedural Fluency (PF) scale
Pre-test	$\alpha = .588$	$\alpha = .613$	$\alpha = .231$	$\alpha = .581$	$\alpha = .646$
Post-test	$\alpha = .827$	$\alpha = .695$	$\alpha = .530$	$\alpha = .597$	$\alpha = .686$

   Statistical Tests. The one-way between-groups ANCOVA was employed to compare the post-test scores of children in the intervention and comparison classes across the five scales,

while controlling for the covariate (the pre-test scores). A preliminary analysis was conducted to ascertain whether the ANCOVA's assumptions of normality, linearity, homogeneity of variances, homogeneity of regression slopes, and reliable measurement of the covariate were violated (Field, 2009). While the assumptions of variance and homogeneity of regression slopes were not violated ($p > .05$) and while the measurement of most covariates was largely moderate (the average being $\alpha = .531$), the results of the Kolmogorov-Smirnov test suggested the assumption of normality was violated across all five scales ($p < .05$). Despite attempts to transform the data using Log transformation, the assumption of normality was still violated. Additionally, the linear relationships between the five dependent variables and their corresponding covariates were found, on average, to be small (the smallest being $R_2 = .19$, and the largest being $R_2 = .43$). Thus, the results should again be treated with cautions.

Results

As previously discussed, in the context of this study, children's conceptual understanding was measured by the score of the OR scale, which is an aggregated score of the CR, VR and SR scales, and thus has a maximum score of 12 points. The higher the score, the more conceptual understanding is evident. Also as previously explained, high Procedural Fluency (PF) scores do not necessarily imply a high degree of conceptual understanding for while some children are able to solve the arithmetic problems accurately, they may not be able to make connections between the different representations. In such example, the score would simply reflect their procedural fluency. The descriptive statistics (see Table 4) show that the Year 4 children across the cohorts scored just over half of the available OR scale on the pre-test. This suggests that their conceptual understanding of multiplication was still relatively weak before the intervention. Similarly, they were able to score just over half of the total PF score.

The non-parametric independent samples Mann-Whitney U test was performed on the five pre-test variables (the PF, OR, CR, VR and SR scales) to assess whether children in both cohorts performed at the same level before the intervention period. The results show that there was no statistically significant difference in children's performance on the PF scale ($U = 333.50$, $p < .939$); on the OR scale ($U = 265.00$, $p < .179$), on the VR scale ($U = 274.50$, $p < .212$); on the SR scale ($U = 275.50$, $p < .228$) and on the CR scale ($U = 302.00$, $p < .502$), suggesting that this was a fair test.

Table 4
Descriptive statistics summarising pre- and post-test data between the intervention and comparison classes

	Pre-test		Post-test	
	Intervention (N = 27)	Comparison (N = 25)	Intervention (N = 27)	Comparison (N = 25)
	M (SD)	M (SD)	M (SD)	M (SD)
PF Scale *(6 points max.)*	4.04 (1.09)	4.00 (1.61)	4.56 (1.42)	3.68 (1.44)
OR Scale *(12 points max.)*	6.15 (2.03)	7.00 (2.52)	7.59 (2.94)	6.24 (3.06)
SR Scale *(4 points max.)*	2.22 (.89)	2.56 (1.19)	3.07 (.96)	2.44 (1.16)
VR Scale *(4 points max.)*	2.59 (.75)	2.88 (.83)	2.78 (1.09)	2.48 (1.05)
CR Scale *(4 points max.)*	1.33 (1.21)	1.56 (1.23)	1.74 (1.46)	1.32 (1.22)

The one-way between-groups ANCOVA was subsequently conducted to compare the post-test scores of children in the intervention and comparison classes across the five scales, while controlling for the covariate (scores of pre-test scales).

Additionally, to illustrate some of the key findings, written response of two lower attaining children in the intervention class, Darren and Sarah (pseudonyms), will also be reported. These two children were chosen primarily because their written responses not only help to highlight areas of their understanding of multiplication that were improved, but also areas where, despite the intervention, they still struggled with.

Procedural Fluency (PF) Scale

After controlling for the covariate, the analysis found a significant difference with a large effect size ($F[1, 49] = 8.19$, $p < .006$, $\eta^2 = .14$) on the post-test PF score between children in the intervention class (adj $M = 4.54$, $SE = .21$) and the comparison class (adj $M = 3.69$, $SE = .21$). Using the adjusted mean scores, children in the intervention class outperformed their peers in the comparison class by 0.85 points on the PF scale. This represents a substantial difference given the maximum score on the PF scale is only 6 points.

Such improvement in procedural fluency of children in the intervention class can be best illustrated by examining Darren's and Sarah's written responses (see Table 5). While Darren's and Sarah's answer to the pre-test Item 5 *(a diagram of 5 groups of 8 triangles)* were 80 and 390 respectively, they were able to provide a correct answer (36) to the post-test Item 5 *(a diagram of 6 groups of 6 triangles)*.

Another marked development can be found in Sarah's response to the pre- and post-test Item 3. While her responses to the pre-test Item 3 and post-test Item 3 were both wrong (i.e. 14 and 48 respectively when they should have been 40 and 54 respectively), it highlighted a marked shift for Sarah from having mistakenly interpreted a multiplicative word problem (10 x 4) as an additive problem (10 + 4) to her being able to recognise the multiplicative relationship between the given numbers in the post-item item. More on this will be discussed in the SR section below.

Table 5

Darren's and Sarah's written responses on some of the items forming the Procedural Knowledge (PF) scale

	Pre	**Post**
Item 3	*Ten children are playing tennis together. Each of them brings four balls. How many balls do they have in total?*	*A town has nine schools. In each school, there are six teachers. How many teachers do the schools have in total?*
	Darren: **40**	Darren: **54**
	Sarah: 14	Sarah: 48
Item 5	*5 groups of 8 triangles*	*6 groups of 6 triangles*
	Darren: 80	Darren: **36**
	Sarah: 390	Sarah: **36**

Bolden written answers were considered as appropriate answers.

Overall Representation (OR) Scale

After controlling for the covariate, the analysis found a significant difference with a large effect size ($F[1, 49] = 13.54$ $p < .001$, $\eta^2 = .22$) on the post-test OR score between children in the intervention class (adj $M = 7.98$, $SE = .40$) and the comparison class (adj $M = 5.82$, $SE = .42$). Using the adjusted mean scores, children in the intervention class outperformed their peers in the comparison class by 2.16 points on the OR scale. This represents a substantial difference given the maximum score on the OR scale is only 12 points.

In the context of this study, it would thus be argued that intervention class children's conceptual understanding of multiplication was significantly higher than that of their peers in the comparison class after the intervention.

In the following sections, findings on the three separate scales (SR, CR and VR) that formed the aggregated OR scale will be examined in turn, and supported by further examples of Darren's and Sarah's written responses.

Symbolic Representation (SR) Scale

After controlling for the covariate, the analysis found a significant difference with a large effect size ($F[1, 49] = 14.32, p < .000, \eta^2 = .23$) on the post-test SR score between children in the intervention class (adj $M = 3.18$, $SE = .16$) and the comparison class (adj $M = 2.32$, $SE = .16$). Using the adjusted mean scores, children in the intervention class outperformed their peers in the comparison class by 0.86 points on the SR scale. This represents a substantial difference given the maximum score on the PF scale is only 4 points.

Such marked improvement can be illustrated by examples of Darren's and Sarah's written responses (see Table 6), particularly in relation to Item 5. For the pre-test, Darren and Sarah chose to symbolically represent a diagram showing 5 groups of 8 triangles with the 40 x 2 and 350 + 40 number sentences respectively. For the post-test, both recognized the multiplicative nature of the diagram and were able to come up with the correct number sentence (6 x 6). Another example of improvement in children's symbolic representation ability can be found in Sarah's written responses to Item 3. For the pre-test, she associated the word problem with addition (10 + 4). For the post-test, while the numbers she used in her number sentence were unfortunately not related to the given word problem, her multiplicative thinking was evident (12 x 4).

Table 6
Darren's and Sarah's written responses on some of the items forming the Symbolic Representation (SR) scale

	Pre	**Post**
Item 3	*Ten children are playing tennis together. Each of them brings four balls. How many balls do they have in total?*	*A town has nine schools. In each school, there are six teachers. How many teachers do the schools have in total?*
	Darren: **10 x 4**	Darren: **9 x 6**

	Sarah: 10 + 4	Sarah: 12 x 4
Item 5	*5 groups of 8 triangles*	*6 groups of 6 triangles*
	Darren: 40 x 2	Darren: **6 x 6**
	Sarah: 350 + 40	Sarah: **6 x 6**

Bolden written answers were considered as appropriate answers.

Contextual Representation (CR) Scale

After controlling for the covariate, the analysis found no significant differences (F[1, 49] = 2.54, $p < .117$, $\eta^2 = .05$) on the post-test CR score between children in the intervention class (adj $M = 1.79$, $SE = 2.3$) and the comparison class (adj $M = 1.26$, $SE = .24$).

Darren's and Sarah's written responses (see Table 7) offer an interesting insight. For example, in relation to Item 1, the word problems that both Darren and Sarah pose to represent the 4 x 7 number sentence bore no relationship to the original number sentence at all, be it in terms of the numerals used (5 and 18 by Darren and 20 and 13 by Sarah) or the mathematical operation adopted (both children used subtraction). For the post-test, while Darren has grasped the concept, Sarah still struggled to pose a word problem that reflect the multiplicative relationship between the numbers.

Table 7
Darren's and Sarah's written responses on some of the items forming the Contextual Representation (CR) scale

	Pre	**Post**
Item 1	*4 x 7*	*3 x 8*
	Darren: Downing has 5p and he spent 18. How much does he have left?	Darren: **There were eight footballs. Three lots of eight came. How many footballs are there?**

	Sarah: Jane was looking in the box of old stuff in the loft. She found 20 stars in the box when she came back half an hour ago she lost 13. How many are there now?	Sarah: One morning, Caylin was working at a café. She had 3 boxes of candy floss and 8 boxes of cupcakes. Can you help Caylin how many candy floss and cupcakes there are in total?
Item 5	*5 groups of 8 triangles*	*6 groups of 6 triangles*
	Darren: Downing has 40p and his friend has 40p more. How much does his friend have?	Darren: **There are 6 schools and 6 teachers in each school. How many teachers are there altogether?**
	Sarah: Joseph has found £350 in his bank account. His lawyer gave him 40 pounds more. How much pounds does he have?	Sarah: Melody has 36 donuts and she eats 12 of them. How many donuts does she have now?

Bolden written answers were considered as appropriate answers.

Visual Representation (VR) Scale

After controlling for the covariate, the analysis found no significant differences ($F[1, 49] = 3.04$, $p < .088$, $\eta^2 = .06$) on the post-test VR score between children in the intervention class (adj $M = 2.86$, $SE = .19$) and the comparison class (adj $M = 2.39$, $SE = .19$).

When examining Darren's and Sarah's written responses (see Table 8), it became apparent they were largely able to visually represent number sentences and word problems during both pre- and post-tests. In fact, as shown in Table 4, children across the two classes performed best on the VR scale, when compared to their performance on the other two representation scales. Subsequently, it may be argued that since the children in both cohorts were already performing highly on the VR scale, the intervention did not necessarily add much to their ability to represent multiplicative number sentences and word problems visually. An exception was Sarah's visual representation of the post-test Item 3. While Sarah recognized the multiplicative relationship between the numbers found in the word problem, why she decided

to then represent 6 teachers in each of the 9 schools with an array of 4 x 12 objects is not apparent. Had it been an array of 6 x 10 or 7 x 9 objects, simple slips in counting might have been the cause. This highlights an area for improvement in future studies to also include interviews where children can be asked to explain their (incorrect) representations.

Table 8
Darren's and Sarah's written responses on some of the items forming the Visual Representation (SR) scale

	Pre	Post
Item 1	*4 x 7*	*3 x 8*
	Darren: **4 groups of 7 circles**	Darren: **8 groups of 3 circles**
	Sarah: **7 groups of 4 stars**	Sarah: **3 groups of 8 circles**
Item 3	*Ten children are playing tennis together. Each of them brings four balls. How many balls do they have in total?*	*A town has nine schools. In each school, there are six teachers. How many teachers do the schools have in total?*
	Darren: **An array of 4 x 10 circles**	Darren: **An array of 9 x 6 circles**
	Sarah: **10 groups of 4 circles**	Sarah: An array of 4 x 12 stars

Bolden written answers were considered as appropriate answers.

Discussion

The current study set out to explore the effectiveness of a short-term intervention whereby 8-9 years old children produced their own multiplication-related picture book to consolidate their conceptual understanding of the topic. Despite the fact that the intervention lasted for just one week, the overall positive findings – both in terms of *procedural fluency* (as measured using the Procedural Fluency [PF] scale) and *conceptual understanding* (as measured using the Overall Representation [OR] scale) – hint at the pedagogical benefits of developing

children's conceptual understanding in mathematics through producing mathematical picture books.

More specifically, the fact that children in the intervention class significantly outperformed their peers in the comparison class on the Symbolic Representation (SR) scale is very promising, particularly when the intervention lasted for just one week. The results confirm the findings of other studies (e.g. Hong, 1996; Jennings et al., 1992; Young-Loveridge, 2004) that found cognitive benefits of using picture books in mathematics learning, though in a slightly different way. More specifically, while children in these studies were treated as *consumers* of mathematical picture books, the children in the current study took on the role of the producers to help them learn, in line with Papert (1991) theory of constructionism.

The Contextual Representation (CR) scale result resonates with the finding of Haylock and Cockburn's (2013) study which highlighted the difficulty young children experienced when trying to translate multiplicative number sentences into word problems. That said, the result was somewhat unexpected. Arguably, the very act of embedding mathematical concepts in a meaningful narrative or story should have enhanced these children's problem posing skills, and thus one might expect their performance on the CR scale to significantly improve as a result of the intervention. One conjecture is to do with the potential role of children's language proficiency and how this may affect their ability to pose written word problems correctly, particularly in relation to using appropriate mathematical language or *mathematics register*, defined by Halliday (1978) as "the meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself)" (p. 195). In the context of this study, an example is when a child represents a diagram of six groups of six triangles (post-test Item 5) using the following word problem "There is 6 packages and 6 giant TVs. How many TVs will there be?" As there was no indication that *each* of the six packages contains 6 TVs, one could very well argue that the total number of TVs remains as six. Indeed,

through the moderation process as discussed earlier, it was agreed by the moderators that a lack of 'each' in this type of situation would mean no point given. Additionally, as it has been well documented elsewhere (e.g. Abedi & Lord, 2001; Martiniello, 2008), children's general language ability (e.g. vocabulary knowledge) is closely tied to their ability to solve word problems. In line with that argument, children's ability to pose word problems could be affected by their language ability. This thus demonstrates how an inappropriate use of mathematical language or *mathematics register* and how a limited general language proficiency could potentially distort children's ability to pose mathematical word problems, and hence children's performance on the CR scale. The very fact that the intervention class children found it difficult to translate number sentences into word problems (CR), but performed much better when translating word problems into number sentences (SR) whereby no written words were required seemed to support this theory. Thus, the provision of opportunities for children to practise posing word problems and to discuss the nature of their problems with their peers and teacher could help develop their contextual representation ability.

Implications

Statistical significance is not the same as practical significance. However, the differences on the PF and OR scales between the two classes represent substantial differences when taking into account of the scales' maximum scores. The promising findings found in this study warrant attention from fellow researchers, practitioners and policy makers, for such intervention can offer an easy-to-implement approach to mathematical instruction and curriculum design. However, more research is first needed to replicate the study to include more children (to avoid making Type II error) and from a wider range of school settings and age cohorts (to increase the generalizability of the findings).

Limitations

The researcher fully acknowledges a number of limitations with this intervention design, particularly the potential bias in the way that children in the intervention cohort could be advantaged through learning multiplication in a way that encourages multiple representations, while their counterparts in the comparison cohort might not necessarily have the same level of opportunity to make those mathematical representational connections. An alternative research design could have been to include an additional class where children get to learn multiplication and to represent it in different ways using a different learning and teaching approach. However, what the researcher would like to achieve with this study's chosen design is to compare and contrast the effectiveness of its recommended approach (i.e. learning mathematics through creating a picture book) with the reality of everyday mathematics teaching and learning. If it was found that the intervention class children's conceptual understanding of multiplication is higher than that of their peers in the comparison class, it would not be to say that the suggested approach is the best approach. What it would demonstrate is simply that the recommended approach is more effective than learning and teaching multiplication in a traditional way. Additionally, the researcher also acknowledges that the intervention period could have lasted longer. Such a short data collection period was largely dictated by the request of the participating school which was under pressure to deliver the curriculum in the limited time that they had. This is not uncommon for schools in England, and it highlights the complexity and limitation of working with schools in a real world context.

References

Abedi, J. & Lord, C. (2001). The language factor in mathematics tests. *Applied Measurement in Education, 14*, 219–234.

Ackermann, E. (2001). *Piaget's constructivism, Papert's constructionism: What's the difference?* Cambridge, MA: Future of Learning Group.

Barmby, P., Harries, T., Higgins, S. & Suggate, J. (2007, July). How can we assess mathematical understanding? In J.-H. Woo, H.-C. Lew, K.-S. Park & D.-Y. Seo (Eds.), *Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education* (pp. 41-48). Seoul, Korea: PME.

Barmby, P., Harries, T., Higgins, S. & Suggate, J. (2009). The array representation and primary children's understanding and reasoning in multiplication. *Educational Studies in Mathematics, 70*, 217-241.

Billings, E. M. H., & Beckmann, C. E. (2005). Children's literature: A motivating context to explore functions. *Mathematics Teaching in the Middle School, 10(9),* 470-478.

Bruner, J. (1966). *Towards a theory of instruction.* Cambridge, MA: Belkapp Press.

Calvert, P. (2006). *Multiplying menace: The revenge of Rumpelstiltskin.* Watertown, MA: Charlesbridge.

Dehaene, S. (1997). *The number sense: How the mind creates mathematics.* New York, NY: Oxford University Press.

Department for Education (2013). *Mathematics programmes of study: Key Stages 1 and 2 National Curriculum in England.* London, UK: DfE.

Department for Education and Employment (1999). *The National Curriculum: Handbook for primary teachers in England.* London, UK: DfEE.

Dienes, Z. P. (1973). *The six stages in the process of learning mathematics.* Slough, UK: NFER.

Elia, I., van den Heuvel-Panhuizen, M. & Georgiou, A. (2010). The role of pictures in picture books on children's cognitive engagement with mathematics. *European Early Childhood Education Research Journal, 18*(3), 275-297.

Field, A. (2009). *Discovering statistics using SPSS.* London, UK: SAGE.

Greer, B. (1992). Multiplication and division as models of situations. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 276-295). New York, NY: Macmillan.

Halliday, M. A. K. (1978). Sociolinguistics aspects of mathematical education. In M. Halliday (Ed.), *The social interpretation of language and meaning* (pp. 194-204). London, UK: University Park Press.

Harries, T. & Barmby, P. (2006, November). Representing multiplication. In D. Hewitt (Ed.), *Proceedings of the British Society for Research into Learning Mathematics* (pp. 25-30). Birmingham, UK: BSRLM.

Harries, T. & Barmby, P. (2007). Representing and understanding multiplication. *Research in Mathematics Education, 9*(1), 33-45.

Haylock, D. (1982). Understanding in mathematics making connections. *Mathematics Teaching, 98*, 54-56.

Haylock, D. (1984). A mathematical think board. *Mathematics Teaching, 108*, 4-5.

Haylock, D. & Cockburn, A. (2013). *Understanding mathematics for young children.* London, UK: SAGE.

Hiebert, J. & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65-97). New York, NY: Macmillan.

Hong, H. (1996). Effects of mathematics learning through children's literature on math achievement and dispositional outcomes. *Early Childhood Research Quarterly, 11*, 477-494.

Janvier, C., Girardon, C. & Morand, J. (1993). Mathematical symbols and representations. In P. S. Wilson (Ed.), *Research ideas for the classroom: High school mathematics* (pp. 79-102). Reston, VA: National Council of Teachers of Mathematics.

Jennings, C. M., Jennings, J. E., Richey, J. & Dixon-Krauss, L. (1992). Increasing interest and achievement in mathematics through children's literature. *Early Childhood Research Quarterly*, *7*, 263-276.

Lamb, E. (2015). Teaching the controversy: Is 5 x 3 five 3s or three 5s? *Scientific American.* Retrieved from blogs.scientificamerican.com/roots-of-unity/teaching-the-controversy-is-5-3-five-3s-or-three-5s/

Lane, S. (1980). *The art of Maurice Sendak.* London, UK: Bodley Head.

Kilpatrick, J., Swafford, J. & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Loreen, L. (1995). *2 x 2 = Boo!* New York, NY: Holiday House.

Martiniello, M. (2008). Language and the performance of English language learners in math word problems. *Harvard Educational Review, 78*(2), 333-368.

Ofsted (2014). *[Anonymised] Primary school report.* Manchester, UK: Ofsted.

Pape, S. J. & Tchoshanov, M. A. (2001). The role of representation(s) in developing mathematical understanding. *Theory into Practice, 40*(2), 118-127.

Ramos-Christian, V., Schleser, R. & Varn, M. E. (2008). Math fluency: Accuracy versus speed in preoperational and concrete operational first and second grade children. *Early Childhood Education Journal, 35*, 543-549.

Skemp, R. R. (1991). *Mathematics in the primary school*. London, UK: Routledge.

van den Heuvel-Panhuizen, M., Elia, I. & Robitzsch, A. (2016). Effects of reading picture books on kindergartners' mathematics performance. *Educational Psychology, 36*(2), 323-346.

Waugh, D., Neaum, S. & Waugh, R. (2013). *Children's literature in primary schools*. London, UK: SAGE.

Yoong, W. W. (1999). *Multi-model approach of teaching mathematics in a technological age*. Paper presented at the 8th South East Asian Conference on Mathematics Education, Manila, the Philippines, 30th May – 4th June 1999.

Young-Loveridge, J. M. (2004). Effects on early numeracy of a program using number books and games. *Early Childhood Research Quarterly, 19*, 82-98.