THE 2022 YOUNG MATHEMATICAL STORY AUTHOR (YMSA) COMPETITION

THE CINDY NEUSCHWANDER AWARD

(THE 12-15 YEARS OLD CATEGORY)

LONGLISTED

'The Journey of Finding Beauty' by Sunny Wang (12 years old)

at Dulwich College Beijing (China)

You can read the author's inspiration for the story and the judges' comments on:

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#YMSAMaths

The Journey Of Finding Beauty

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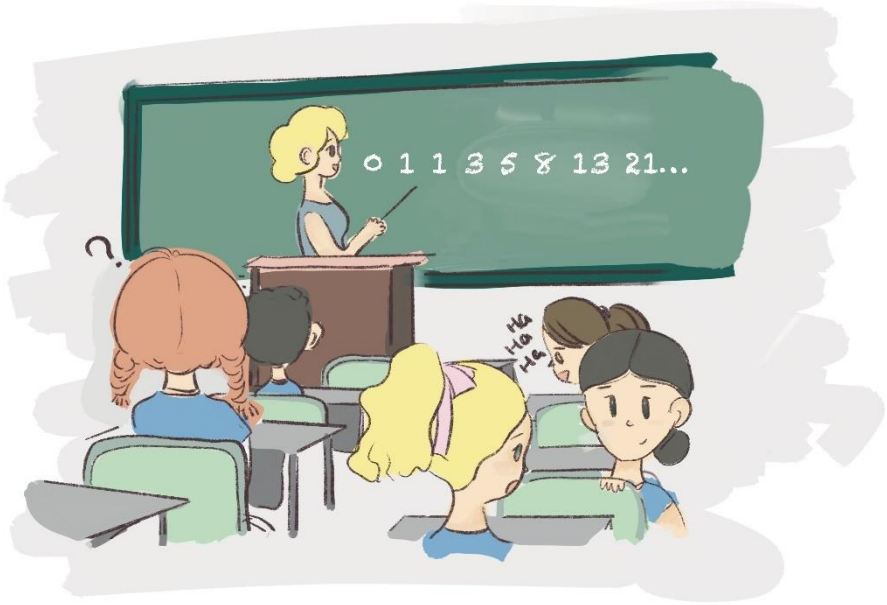


"Lastly, I will show you this magic set of numbers.", said Ms. Kwan, who was the math teacher, "It is called The Fibonacci Sequence, which is also named the golden sequence. Using this sequence, you will find how amazing nature is! How to do this? This will be your homework during the summer holiday."

Most of the students were not really listening to Ms. Kwan, because it was the last class before summer holiday. Juliana stared at the whiteboard and thought, Fibonacci Sequence? How can numbers represent beauty? These just look like boring numbers that we usually learn in math class.

Juliana's appearance is plain, her long red hair was braided into two braids. Unlike many adolescent girls who like to get together, Juliana had low self-esteem because she was not good-looking enough and liked to be alone.

"So, if there's no more questions about today's math class, then I'd say it's time to pack up and head home." announced Ms. Kwan.

The students jumped up and ran out of the classroom.

"Have a wonderful holiday break! Don't forget to do your homework! You will find beauty around you using this sequence!" added Ms. Kwan.

Beauty never has anything to do with me, thought Juliana.

Entering her room, Juliana put her bag on the desk and sat down. She looked out from her window and murmured, "what a nice summer day."

She opened the window. Suddenly, an elf fell from the sky and landed on her desk. The little elf said, "Ugh, this is the third time I fell from that chimney, maybe I really should lose some weight..."

Then, the elf saw Juliana, her eyes opened wide. She froze.

Juliana lifted the elf up in the air.

“Who are you?”

“Aye put me down! You are so rude.”

As the elf landed, she patted the dust off her clothes, breathed deeply to make herself calm down, and said, “well well, since you’ve already seen me, let me introduce myself. My name is Ziffa. I am a member of Pythagoreanism.”

“Pythagoreanism? What is that?” asked Juliana.

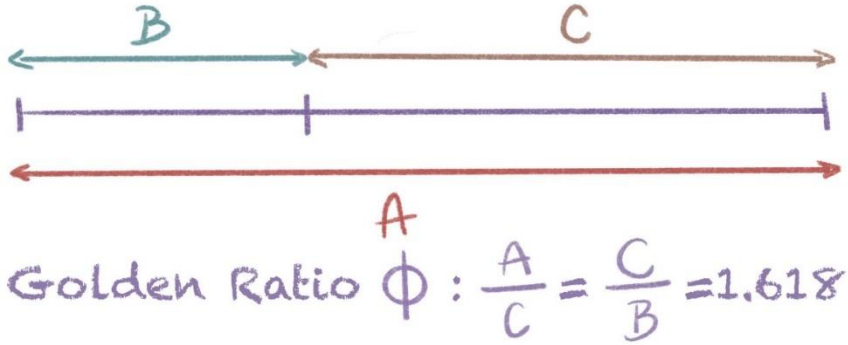
“Do you remember what your math teacher taught you today?” Ziffa asked rhetorically.

“The set of numbers, um… the Fibonacci Sequence?”

“Yes,” Ziffa nodded, “Pythagoreanism and the Fibonacci Sequence have a strong connection. Pythagoreanism believes the mystery of the physical world exists in mathematic harmony, and they were especially interested in the Golden Section.”

“Golden Section? What is that?”

“The Golden Section is well known as the Golden Ratio. In mathematics, the Golden Ratio is an irrational number often denoted by Greek letter ϕ, which is approximately equal to 1.618. It is the ratio of line segments cut into two pieces of different lengths, such that the ratio of the whole segment to that of the longer segment is equal to the ratio of the longer segment to shorter segment.”



“It looks like you're teaching me math.” Juliana yawned.

“I’m teaching you how to find beauty in numbers. Pythagoreanism believes the Golden Ratio provides the most aesthetically pleasing proportion.

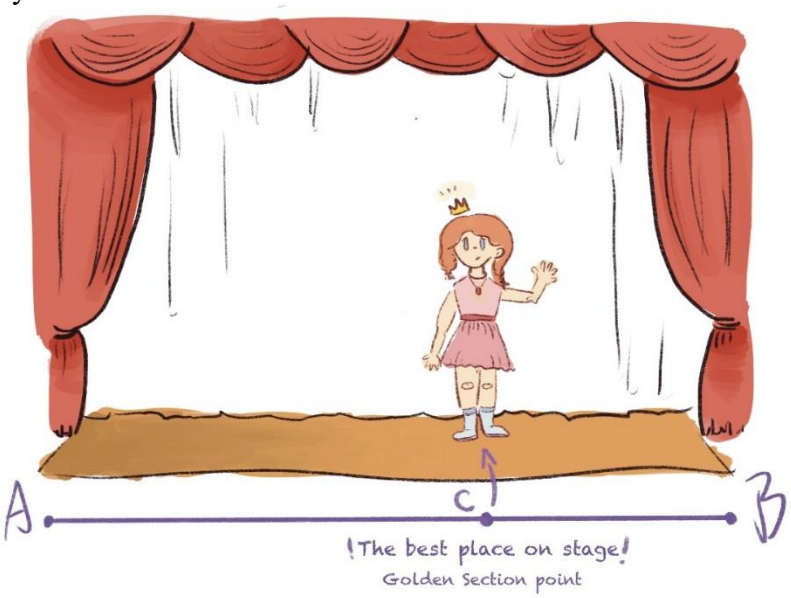
“What does that mean?” Juliana asked curiously.

“Let’s look at an example,” said Ziffa. “The Egyptians made the ratio Between the pyramids’ base and height really close to the Golden Ratio.”

“In the Parthenon in Greece, or modern architecture such as the U.N. Building in New York, you can find the Golden Ratio, also. This Golden Ratio does not only make the architecture beautiful, but also makes it more stable,” Ziffa continued while Juliana was quite shocked. “Actually, artists also use the Golden Ratio in their artworks. It can be seen in *The Last Supper*, the *Mona Lisa*, and the *Virtruvian Man*.”

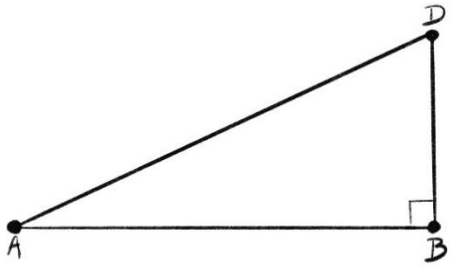
“It is impossible!” Juliana exclaimed totally amazed.

“Let me give you one more example, if you do any speeches on stage, it is quite weird if you just stand in the middle of the stage, but if you stand right where the Golden Ratio point is on stage, then it will make the audiences comfortable while looking at you.”

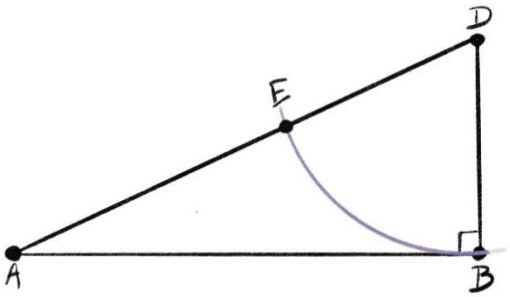


“That’s interesting! So how can I find this perfect point?” Juliana asked.

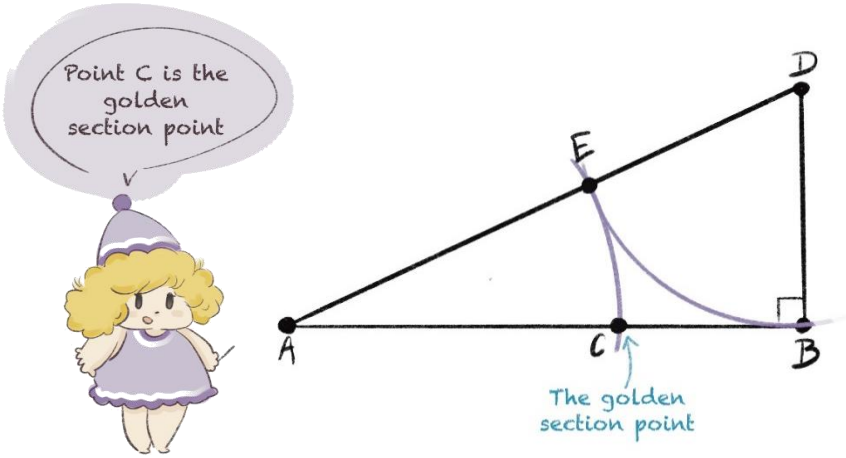
“Good question! There actually is a method to find out. First, we should draw a vertical line off AB from point B, calling this segment BD. The length of BD must be 1/2 the length of AB. So ▲ABD is a right angle triangle.



Now, use point D as the middle point and draw a circle with BD as the radius. See where the circle and line AD cross and label that point E.



Use point A as the middle point and draw another circle using AE as the radius. See where the circle and line AB cross and label that point C.”



“This method is really clever! Let me verify it.” Juliana eagerly said.

if $AB=a$

$BD=\frac{1}{2}AB=\frac{1}{2}a$

Using Pythagoras theorem

$$AD^2=AB^2+BD^2$$
$$=a^2+(\frac{1}{2}a)^2$$
$$=\frac{5}{4}a^2$$

So $AD=\frac{\sqrt{5}}{2}a$

radius $DE=BD=\frac{1}{2}a$

So $AE=\frac{\sqrt{5}}{2}a-\frac{1}{2}a=\frac{\sqrt{5}-1}{2}a$

radius $AC=AE=\frac{\sqrt{5}-1}{2}a$

So $\frac{AB}{AC}=\frac{a}{\frac{\sqrt{5}-1}{2}a}=\frac{2}{\sqrt{5}-1}\approx 1.618$

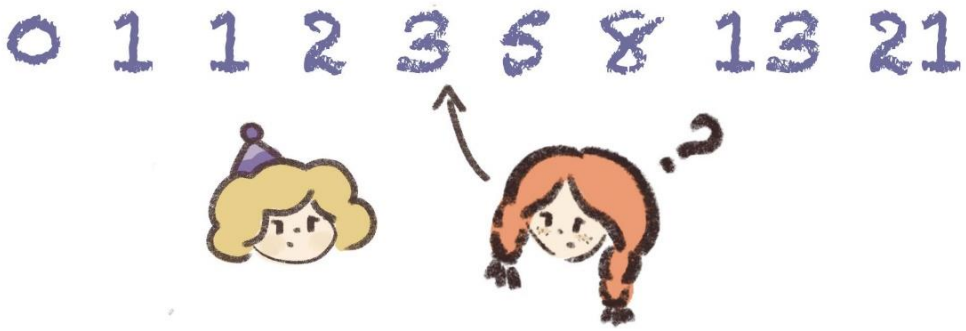
$$\frac{AC}{BC}=\frac{\frac{\sqrt{5}-1}{2}a}{(1-\frac{\sqrt{5}-1}{2})a}=\frac{\sqrt{5}-1}{3-\sqrt{5}}\approx 1.618$$

In conclution $\frac{AB}{AC}=\frac{AC}{BC}\approx 1.618$

So C is the Golden Section point of Segment $\overline{AB}$.

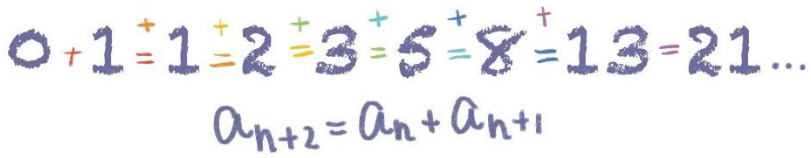
“Excellent! You got it.” Said Ziffa gratified.

“Ah ha, now I understand how Golden Ratio works, but what about the Fibonacci Sequence? What does the Golden Ratio have anything to do with Fibonacci Sequence?”



“Think about it.” Ziffa encouraged Juliana.

Juliana stared at those numbers for a while…wait…she found something."0+1=1,1+1=2,2+3=5…” Juliana started writing quickly. When she finished writing $a_n = a_{n-1} + a_{n-2}$ Ziffa smiled.



“So let me think, maybe the Golden Ratio hides in this sequence.” It seemed Juliana couldn't stop thinking.

“You are getting smarter.” Ziffa applauded.

Juliana tried to write down some numbers which could be found in the Fibonacci Sequence:

A	B	B : A
2	3	1.5
3	5	1.66666...
5	8	1.6
8	13	1.625
13	21	1.6153846...
...	...	...
144	233	1.6180805...
233	377	1.61802575...
...	...	...

“Ziffa, see it? In the Fibonacci Sequence, the ratio between two numbers eventually approaches the Golden Ratio.” Juliana was exited.

“Yes, so that we call Fibonacci Sequence as Golden Sequence.” Ziffa quickly added, ”The Fibonacci Sequence is not only about this, let me take you to see something more interesting, come outside with me.”

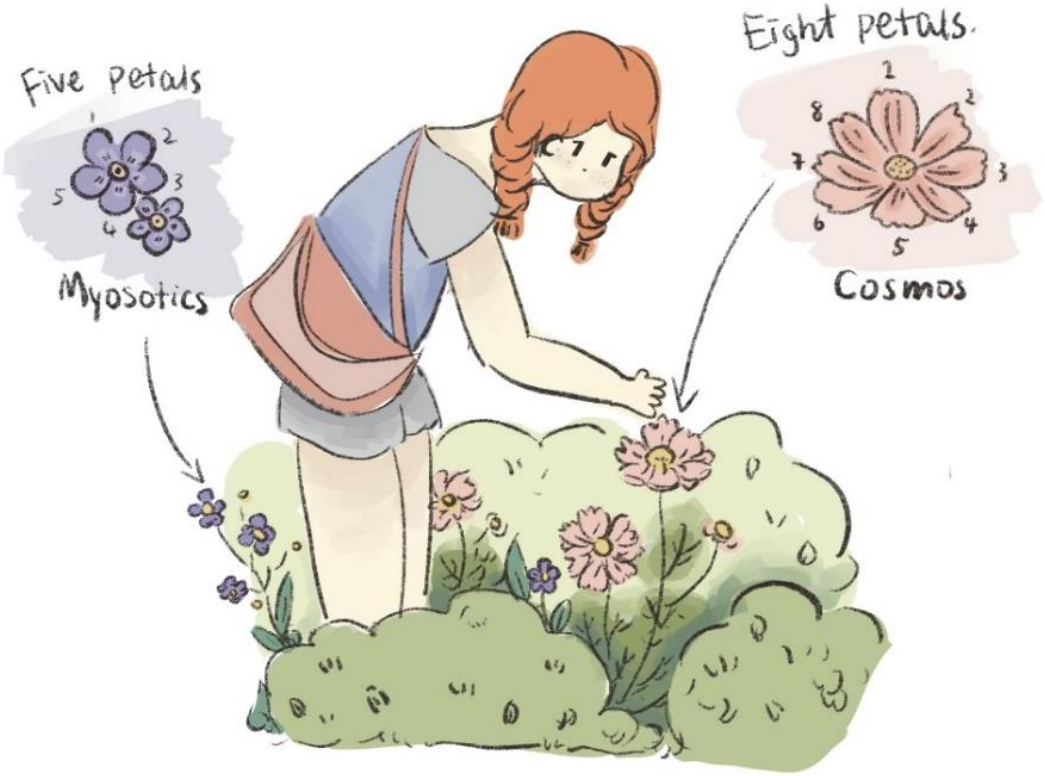
Juliana followed Ziffa out to her garden.

“There are so many flowers in your garden!” Ziffa said, “But have you ever counted the number of petals on those flowers?”

“Count petals?” Juliana looked at the purple flowers at her feet in doubt. “This one has five petals.”

“Its name is myosotis. Try the one beside it.” Ziffa smiled.

Juliana saw there were some pink flowers beside those purple ones, and the number of the petals on the pink flowers was eight.” That’s interesting,” Juliana thought, “5 and 8 are both numbers in the Fibonacci Sequence.”



“These pink flowers are named cosmos. Why don't you even know the flowers in your own garden?”

But Juliana did not hear what Ziffa said. She was busy counting the petals of another flower. “1,2,3…20,21! The daisy has 21 petals, and it is also the number in Fibonacci Sequence!” Juliana shouted.

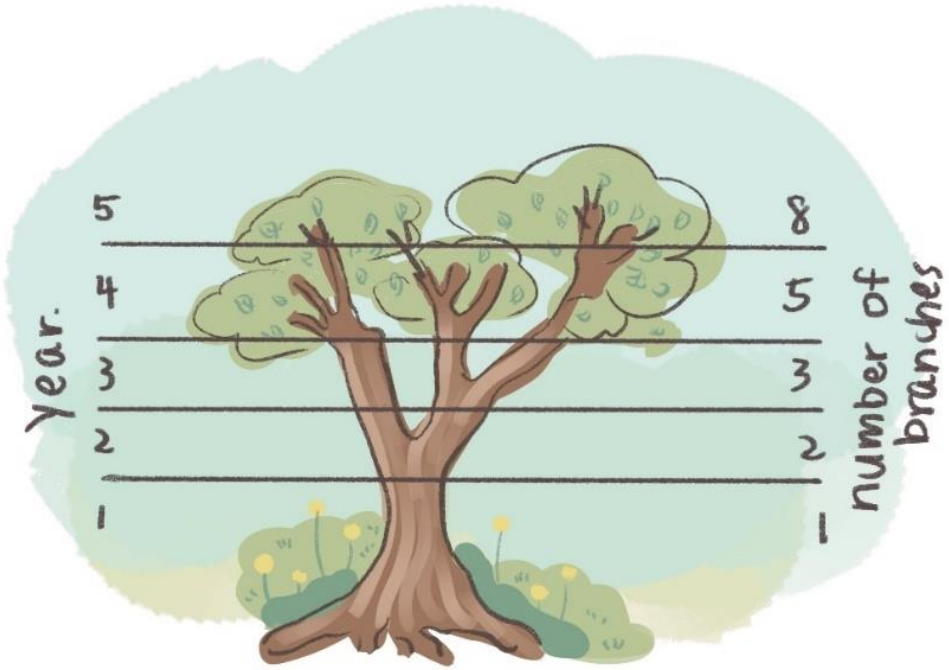
“The Fibonacci Sequence’s presence in the nature is far more than this. If you can see sun flowers, you will find the arrangement of their seeds follows the rule of the Fibonacci Sequence too.” Ziffa seemed proud.

“Why? Did nature also learn mathematics?” Juliana asked.

Ziffa laughed out loud, “Sunflower seeds arranged in this way make the best use of sunlight and air, which can let them breed more offspring.”

“Plants are clever!”

“Yes. See that big tree over there?” Ziffa pointed to a phoenix tree. ”In the growth of a tree, the new branches need a period of time to rest in order to have more energy to grow, then they will germinate new branches after resting. When an old branch A germinates a new branch B, the next year B will rest and old branch A will continue to germinate new branch C. Then the third year, branch C rests and branch A and B continue to germinate new ones. This natural sequence can be shown as:



Juliana’s eyes sparkled, ”It is Fibonacci Sequence again!”

“Yes. How old is this tree?”

“Let me think, 12 years old.”

“Go on and count how many branches does the tree have.”

“1,2,3,5,8,13,21,34,55,89,144,233. Can you believe it? There are 233 branches on my tree!”

“If it hadn't been destroyed by birds, then your tree is also following the Fibonacci Sequence.” Ziffa smiled.

After the excitement, Juliana suddenly became somewhat forlorn. “It seems that plants are smarter than me.”

“Of course not. You proved the Golden Section point, and you found the secret of the Fibonacci Sequence. You are a smart girl.”

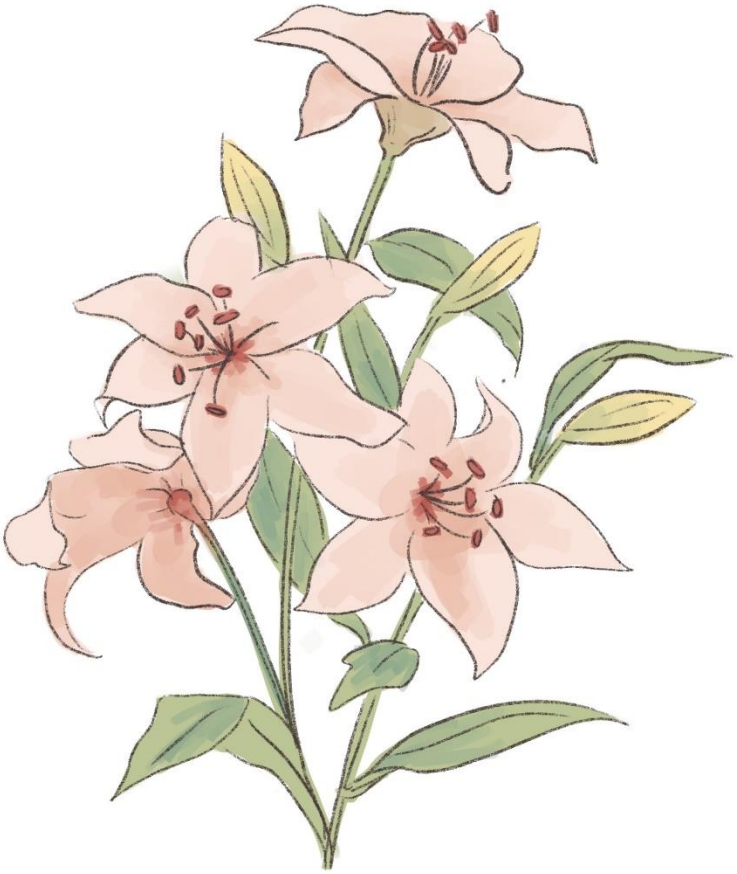
“But those plants look beautiful，because they grow by the Golden Ratio rule. I am not.” Juliana felt depressed.

Ziffa took her to the lily. “Do you think the lily is beautiful?”

“Yes of course, it is my favorite.”

“But the lily has 6 petals, which does not follow the Fibonacci Sequence. Our nature has many rules. The Fibonacci Sequence is only one of them. There are many more things that are beautiful but do not follow the Golden Ratio. Just remember one thing, the golden ratio might be one way to measure beauty, but not the only way. Beauty can be displayed in different ways, so don't get trapped by what we learned today!"

Juliana thought for a moment, then her eyes suddenly be bright, "I understand. The world is more lovely because of different definitions of beauty. Thank you, Ziffa, thank you."



On the stage, Juliana is giving a speech on Fibonacci Sequence to her classmates. Standing at the Golden Section point on the stage, she confidently tells the beauty of mathematics.