THE 2021 YOUNG MATHEMATICAL STORY AUTHOR (YMSA) COMPETITION

THE CINDY NEUSCHWANDER AWARD

(THE 12-15 YEARS OLD CATEGORY)

SHORTLISTED

‘The Jewel of the Tree’ by Rebecca Kear (14 years old)

at Dulwich College Beijing (China)

You can read the author’s inspiration for the story and the judges' comments on:

www.mathsthroughstories.org/ymsa2021

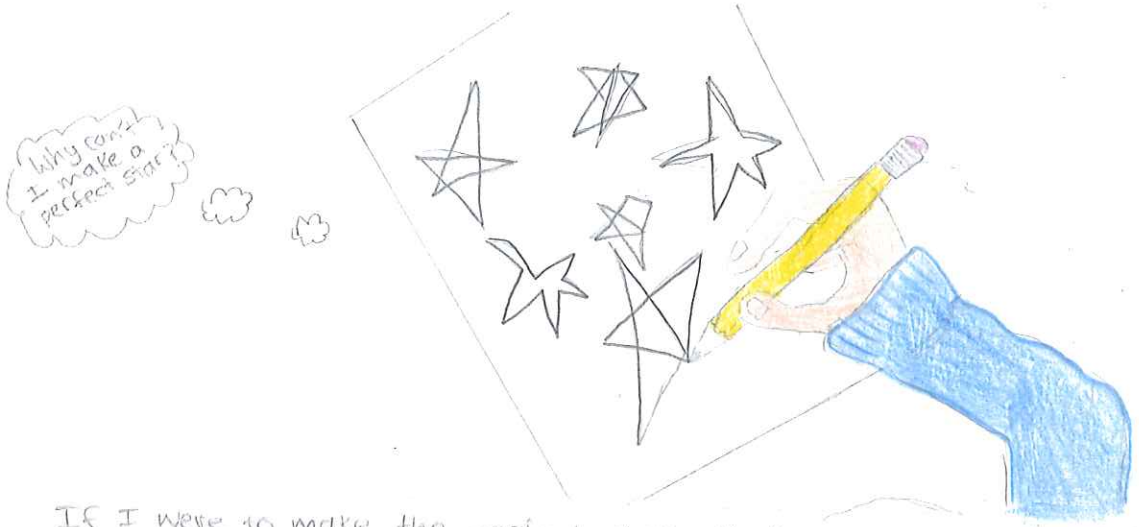
#YMSAMaths

BY REBECCA KEAR
DULWICH COLLEGE BEIJING

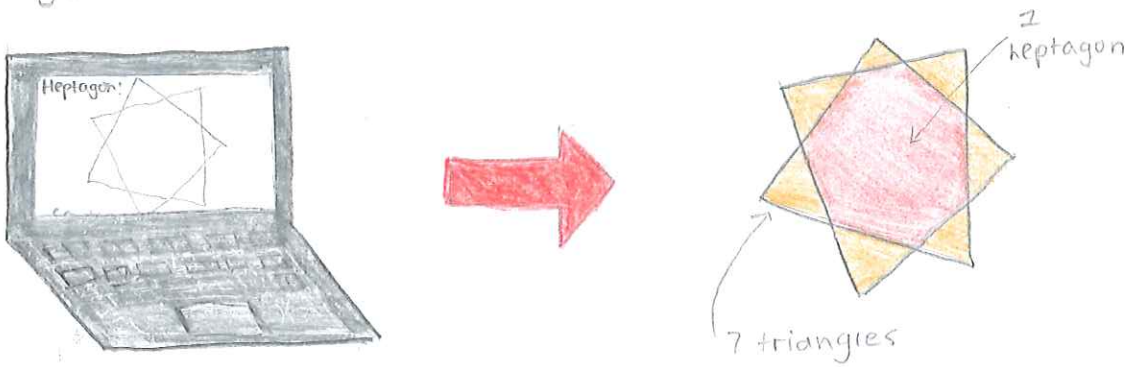
I was out picking holly berries when Nana May hollered from the back porch door, beckoning me to come into the living room with her. It was around Christmas time when it happened with the snow beginning to fall and the chilly weather coming down Dixie Avenue. I could smell bitter-sweet hot chocolate steaming on the kitchen stovetop when Nana May opened the back porch door, letting the wafts of aromatic air make its way to my nose. Mother always said this time of the year was like no other in North Port County when all the streets were filled with lights and festive decorations, and every individual was filled with the utmost joy. The best thing of all was that it snowed each year more than a meter of snow, so I never had to worry about not having a white Christmas.

It was the week before Christmas Day. As the days grew colder, our family was fixing up the final decorations for the tree and was hanging lights, ornaments, and stockings in the living room. Nana May carried me into the living room with her long wooden cane, pointing at the top of the Christmas tree and lecturing me how important it was to have a star on the top - now it was the jewel of the tree. The tree was beautiful; filled with elaborate ornamental pieces, ribbon, and wooden animals my dad carved out. Everything was perfect, excluding the fact there was no star on the top. I knew it was foolish of me not to put the star on, since it was my duty decorating the tree, but I told Grandma I had lost the star last winter when the Ewells, our neighbors, had borrowed it and not returned it. Without saying a word, Nana May handed me some card board, a ruler, a pencil, a protractor and an x-acto knife.

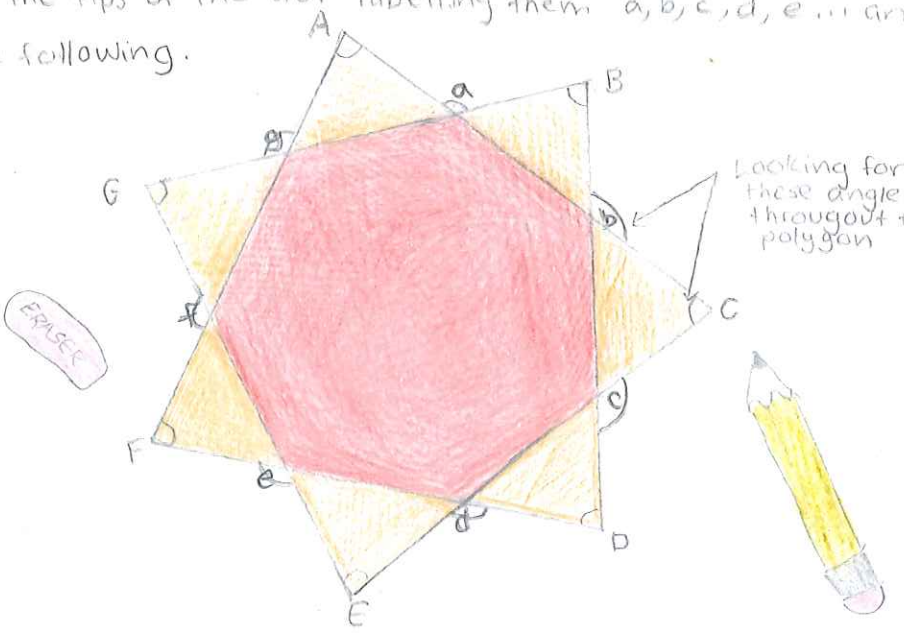
I knew I would have to make a new star for our tree, but I was confused why I would need a protractor to make the shape. I soon realized this after I began my first few attempts to make the star -- a complete mess!



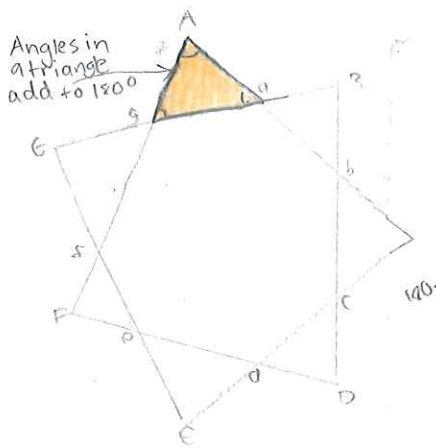
If I were to make the perfect star, all the angles would need to be exact and every edge would need to be the same length. I realized that this star-making process would rely on mathematics and since I wanted to make this star perfect, I decided to use some of the skills I gained in geometry class. I scanned the internet finding the perfect seven-pointed star. I wanted to find all the necessary angles that made up this heptagram in order to make the star. It occurred to me that this star was actually made up of a heptagon - a seven-sided polygon - and seven triangles. I got the following!

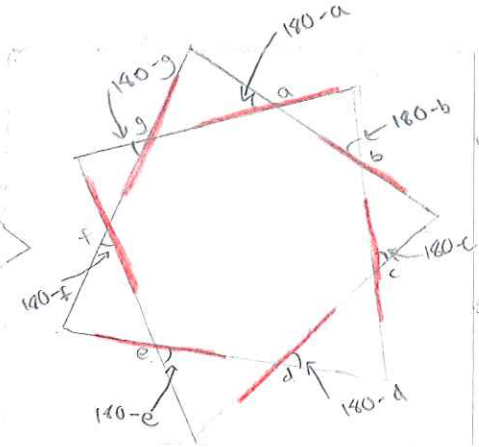


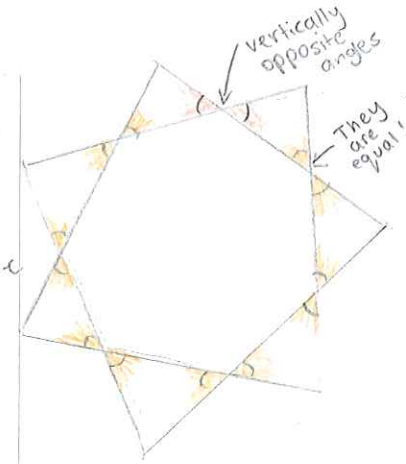
I decided to first find the interior angles of the seven-points of the star labelling such points A, B, C, D, E...., then find the exterior angle connecting the tips of the star labelling them a, b, c, d, e... and so on. I got the following.



To solve for the angles of the seven points of the star, I would need to know the two base angles of each of the triangles, then subtract those from 180 since all angles in a triangle add to 180 degrees. All of the triangles in this heptagram are isosceles meaning the base angles would be the same. I noticed that angles a, b, c, d, e, f and g were placed on a straight with one of base angles of the triangle supplementary to angles a, b, c, d... This means one base angle is equal to 180 - a, 180 - b, 180 - c, ... depending on the location. Since the base angles of these triangles form an "X," two pairs of vertically opposite angles would be created, meaning there would be two angles equal to 180 - a, two angles equal to 180 - b, and so on.







Now that I knew what each base ange should be equal to, I could subtract the two base angles of the triangles to leave the angles of the points of the star. For example, point A equals to 180 minus (base angle plus base angle) which is equal to $180-(180-g+180-a)$, simplifying to $180-(360-(a+g))$ or $a+g-180$. Thus angle A is equal to $a+g-180$. This is likewise for points B, C, D, E, F, and G. So, the sum of all angles of these points would be the following:

Angle A + B + C + D + E + F + G =

$(g+a-180)+(a+b-180)+(b+c-180)+(c+d-180)+(d+e-180)+(e+f-180)+(f+g-180)$

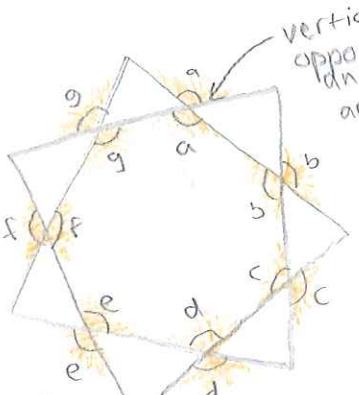
=

$a+a+b+b+c+c+d+d+e+e+f+f+g+g-180-180-180-180-180-180-180$

Ultimately simplyfing to:

$2(a+b+c+d+e+f+g)-7(180)$

It had just occurred to me that the sum of the lower case letters was actually the same as the sum of the angles in the heptagon in the center of the heptagram.



$a+b+c+d+e+f+g$ = sum of angles in heptgon

So this meant that the sum of the points of the star must be equal to, after substituting...

$2(\text{sum of angles in a heptagon})-7(180)$

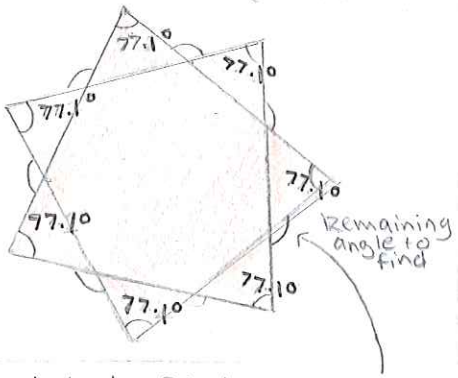
which equals using the formula $(n-2)180$ for any n-sided polygon,

$2(7-2(180))-7(180) = 2(5(180))-7(180) =$

$10(180)-7(180) = 3(180)$ OR 540

I burst out in joy when I finally solved for the sum of the interior angles in the points of the star! 540 degrees. And since I knew there were 7 points on the star, I could divide this by 7 to get the exact angle of each individual point of the star. I rushed upstairs to my bedroom, trying to ignore the clutter and mess, and pulled out my calculator from my desk cabinet, punching in the numbers.

$540/7$
$= 77.1428$
≈ 77.1 (1 dp)



But the work was not over yet. I still needed to find the remaining angles necessary to draw out the star. I had already solved for the angles of each point of the star. It occurred to me that each of the remaining angles I wanted to find were actually equal to one angle of the heptagon. This is due to the fact that the angles I wanted to find was vertically opposite each individual angle of the seven-sided polygon within the star. And since the heptagon is regular each angle is the same.

This meant that I just had to use the following formula:

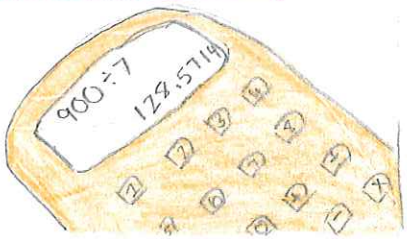
$$\frac{180(n-2)}{n}$$

for each n-sided polygon. I substituted n for 7, getting the following:

$$\frac{180(n-2)}{n} = \frac{180(7-2)}{7} = \frac{180(5)}{7} = \frac{900}{7}$$

Typing into my calculator, I found the exact angle =

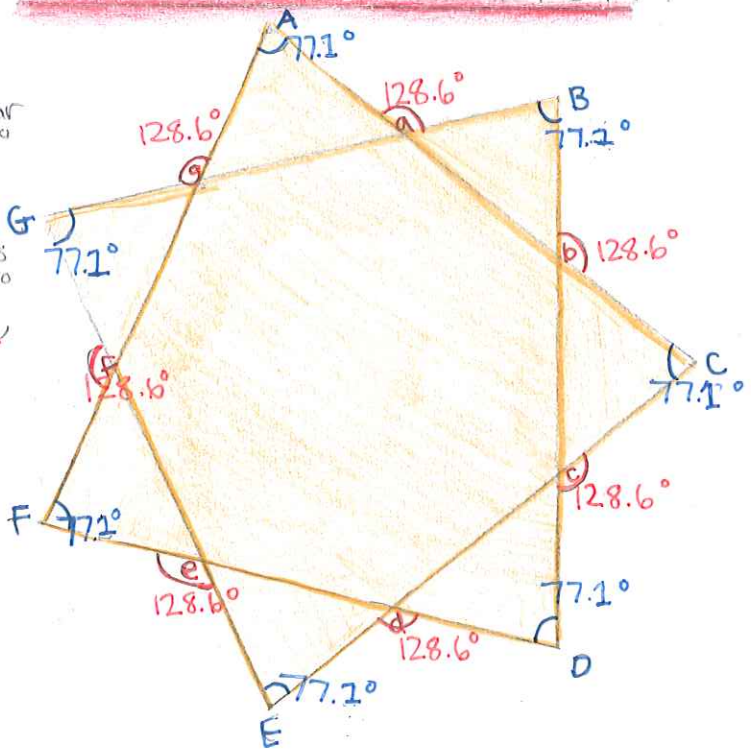
128.5714... or 128.6 (2 dp) degrees



So I now knew the exact angles I would need to make the star! 77.1 degrees for the points of the star and 128.6 degrees between these points, I decided to make each edge 10 centimeters long. My final solution looked like the following:

FINAL SOLUTION

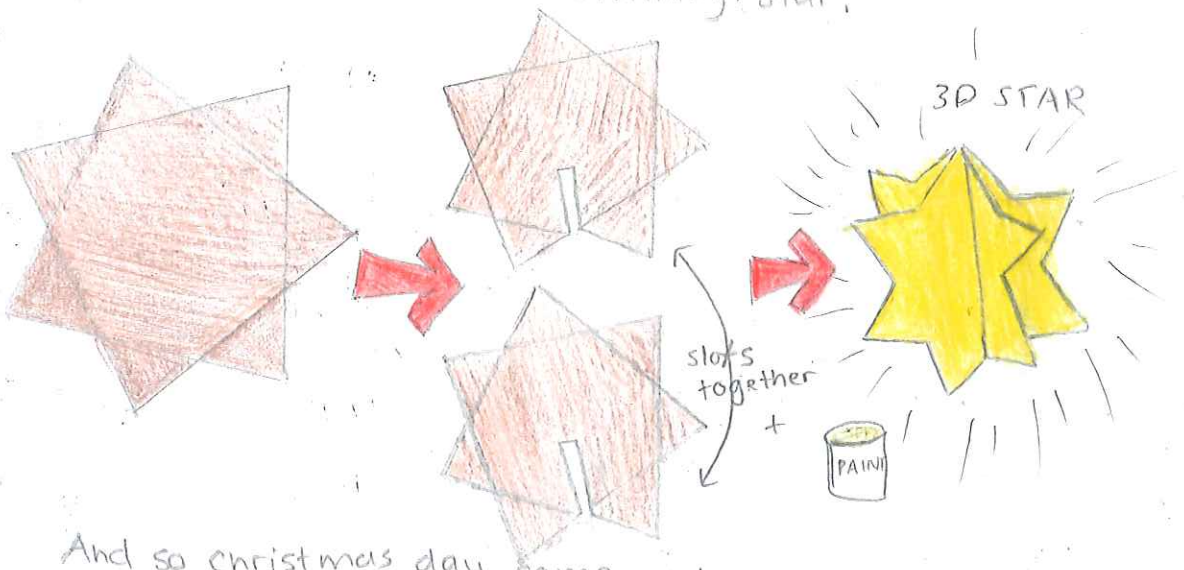
- points of the star are each 77.1°
  - ↳ Angles A, B, C, D, E, F, G
- remaining 7 angles are each 128.6°
  - ↳ Angles a, b, c, d, e, f, g



Uncle Roger, who was spending the week over for Christmas, came into the living room holding a shaver with his beard covered in white shaving foam. He came in and handed me some yellow paint and sparkling glitter, saying, "Here's some paint for that star Nana May was talking to me about. She said to give it to you. And what's taking you so long anyway?" He walked back into the bathroom and I just gave a grin, knowing the time I had spent understanding how I would make this star would result in a stunning star - or as Nana May says- "the jewel of the tree."

I got out the cardboard Granny gave me earlier and, using my protractor, I marked the angles that I knew created this star. I used my ruler to measure out 10 centimeters for each edge. I then used the x-acto knife to cut out the shape.

I duplicated the process, making two cut-out seven-pointed cardboard stars. Using the x-acto knife once again I cut out two slots; one for each star, and fitted the stars together to make a 3D star. I then colored the star with the yellow paint and golden glitter Uncle Roger gave me and created the most stunning star.



And so Christmas day came and was the most spectacular celebration yet. Our whole family- Uncle Roger, Nana May, Mom Dad, Lu-Lu, my younger sister, and I were there feasting on pies, fruit, cake, cookies, and hot coco. It was a sentimental scene as our family gathered under the sparkling tree and tore open the gifts we had for eachother. I smiled as I proudly saw the star I made glimmer beneath the dazzling sunlight beaming into our room. The laughter, smiles, and love added a layer of warmth as I watched snow flakes slowly fall gently upon the ground of New Port County out from the living room window. And so Christmas passed by faster then hoped only for me to lose the star once again for the third year in a row. I least I had a whole year to make a new one!

The End

ABOUT THE AUTHOR

REBECCA KEAR

Rebecca Kear was born in Harbin, China, but is half Korean and half British. She enjoys art, music, and technology classes in her school, Dulwich College Beijing. Rebecca is 14 and enjoys anything from sports such as gymnastics to coding programs. Rebecca combined her love of christmas and mathematics in her story.