THE 2022 YOUNG MATHEMATICAL STORY AUTHOR (YMSA) COMPETITION

THE CINDY NEUSCHWANDER AWARD

(THE 12-15 YEARS OLD CATEGORY)

SHORTLISTED

‘Paper Cranes’ by Audrey Wen (14 years old)

at Dulwich College Beijing (China)

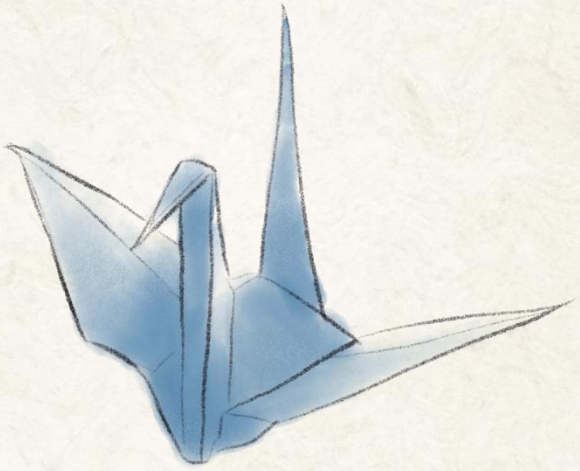
You can read the author’s inspiration for the story and the judges’ comments on:

www.mathsthroughstories.org/ymsa2022

#YMSAMaths

Paper Cranes

Exponential Growth

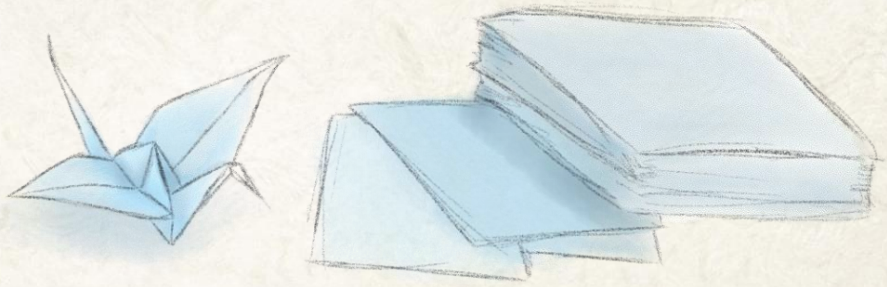


By Audrey Wen DCB 2022

I was working on some origami cranes this afternoon, it was for the decorations I'm bringing to class next week and I'm aiming for 30 finished by the evening. The steps to making them were simple, but because it required me to fold the small piece of paper in half many times, the paper often became too tight and thick for me to fold again and again. At my ninth crane, my sister was home with the sound of the doorbell. Having quite a pair of delicate hands, she was good with art and crafts, so we decided to work on them together at the kitchen table.

"These take quite some effort to perfect," My sister chuckled, "The paper is too stiff to work with after so many folds."

"Yep," I answered, "That is exactly why I asked you to help, to minimize the workload…"



I glanced over at my sister, who was looking out of the window, seeming to be lost in thought. I immediately knew that she has found something worth exploring.

"Speaking of folding paper," My sister began, "How many times do you think you can fold a piece of paper like the ones we're using now, in half?"

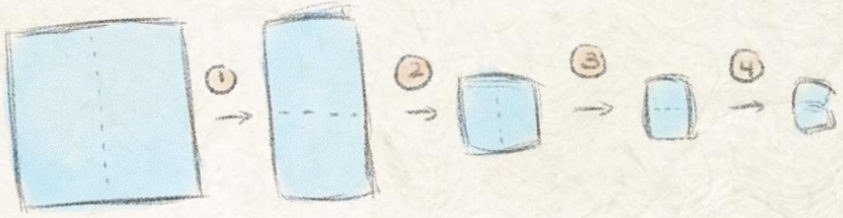
It was a question that seemed pretty clear and easy to answer at first, but thinking back on how difficult it was to even fold the piece of origami paper 3 times confused me.

"15 times?" I guessed quietly. "It would depend on how big it is too, right?"

"Good thinking," She complimented, "But the record was 11 times. Even if they tried folding a sheet of larger size, it was hard to go beyond that number."

It was a fascinating fact to process. Hard to believe too, unless I tried myself. And so I did, with my sister watching as she continued working on our cranes. The paper I used was a little thinner than the thickness of a standard printing paper, just below 1mm. Not very thin for this kind of experiment, but the fact that the record was 11 times was too incredible for me to think that this bare thickness would make any difference. To my surprise, I was already struggling at my fourth fold, and my fingers were hurting from how hard the folded paper has become.

"I admit it is quite impossible to go further than this…" I laughed at myself. My sister was suppressing a smile too.



"Now, assuming that it is possible, how many times do we need to fold a piece of paper for the thickness to become – about how tall I am?" She asked again.

"My guess would be… 15 folds." I replied after a while of thought, which was quite pointless because I couldn't seem to figure out anything.

"That's a really close guess! It would take about 17 folds. But even one fold more would make a significant difference." Soon enough, my sister noticed that I looked pretty overwhelmed by what she was trying to tell me. "Do you want me to explain, or another example to try?"

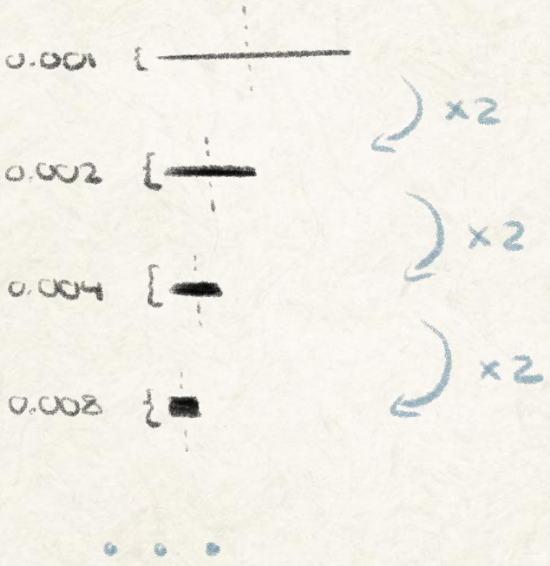
"Another example, please." I replied eagerly. I was determined to work out the answer by myself this time.

"Sure, here is one on an even larger scope." She said, "How many folds do you think it will take for the thickness of the piece of paper folded to become the distance between the earth and moon?"

This time, I stopped estimating and tried to look for some mathematical method to solve this problem, setting pen and paper ready on the table. Noticing that my piece of paper was blank with no ink for quite a while, my sister took over the pen and started to explain for me.

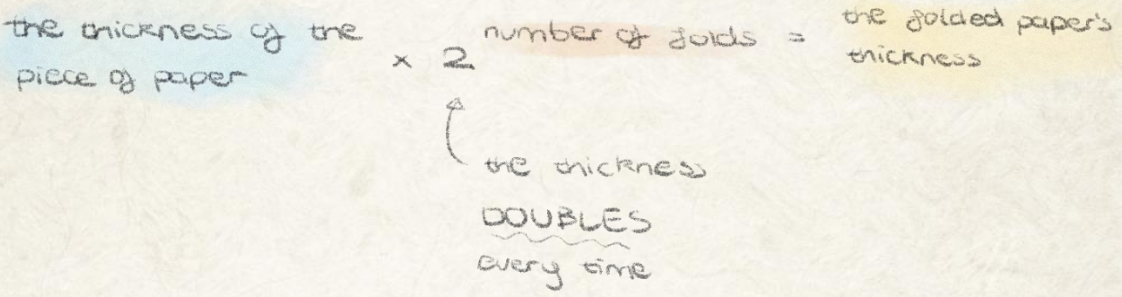
"Listen carefully," she said before she began. "Let's say that the paper we decide to use is 0.001cm in thickness. If we fold it once, the thickness doubles and becomes 0.002cm because two layers of the same paper is being overlapped."

"After we fold it the second time, it becomes 0.004cm thick, after the third fold 0.008cm thick, and so on. Do you notice a pattern with how thick the paper is becoming?"

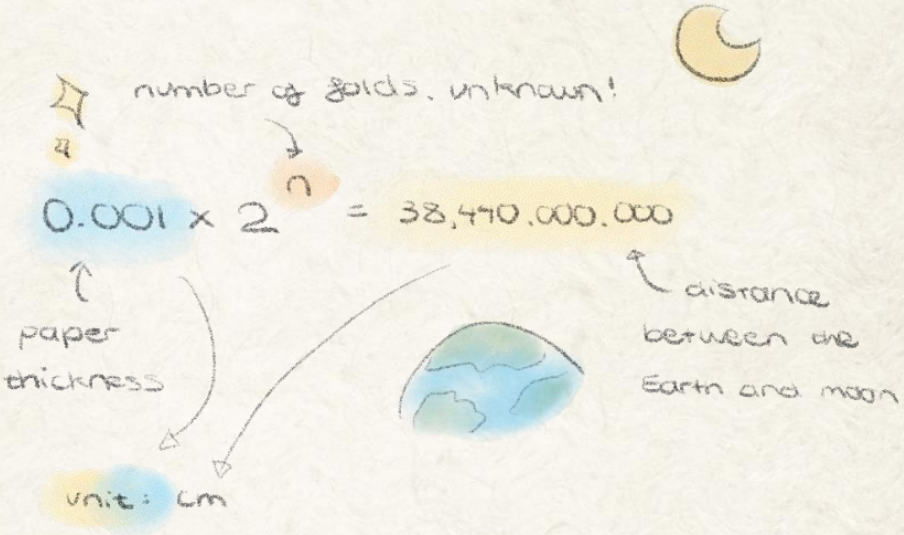


"It doubles every time you fold it, doesn't it?" I pointed out.

"Absolutely." My sister agreed, "And because a pattern exists, we can write down an equation for calculating the thickness of the piece of paper after a certain number of folds. That would be: thickness of the paper, which in the case is 0.001cm, multiplied by 2 to the power of the number of folds."



"Now that we have an equation, we can go back to the question of how many folds would take us from the earth to the moon and substitute our known values into the it. We know that the paper used in this example is 0.001cm. As for the distance between the earth and moon, if you search it up, it is about 384,400 km, which would be equal to 38,440,000,000cm. Using the same unit(cm) will make things much easier."



"Now, do you want to have a go at solving it?" My sister suggested, sliding the paper to me.

I nodded and smiled, taking the pen.

$$0.001 \times 2^n = 38{,}440{,}000{,}000$$

$$2^n = 38{,}440{,}000{,}000 \div 0.001$$

$$2^n = 38{,}440{,}000{,}000{,}000$$

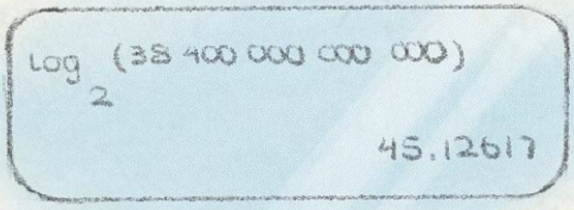
$$n =$$

However, I was stuck when I tried to work out what n was. The number was big so trial and error wasn't an efficient method to get to the answer.

"Stuck?" My sister suddenly asked, as if she could read my mind. "Don't worry about working out n by yourself, I can't even do it and we will be using a calculator for that. It's alright! Now, how about you take a guess first, about how big n is? Tens? Hundreds? Thousands?"

"It is a pretty big number on the right of the equation," I said, "I'd say… a few hundred or so?"

"I would have guessed the same thing until I realized how much just a few powers can make a number grow." My sister was quite excited to share the answer, typing on her calculator now. "But in reality, it only takes about 45 folds."



I gasped.

"Yep. This is how many folds one single piece of paper would take to become as thick as the distance between us and the moon. This is the power of powers." My sister raised her eyebrows at me, smiling as she concluded her miraculous explanation. "What do you think?"

I was searching for words. And that is the power of powers.

"Just… wow." I managed to exclaim. "But… if this really was the case, shouldn't we be building a paper bridge to the moon already? Instead of sitting here making paper cranes, that would be far more interesting. And it would only take 45 folds!"

Hearing my silly words, my sister burst out laughing and I soon joined in too. A paper bridge to the moon would be interesting, I thought, but these happy moments are worth much more.

Have you ever wondered how many times you can fold
a piece of paper for it to be thick enough to get you to the moon?
Let's find out.