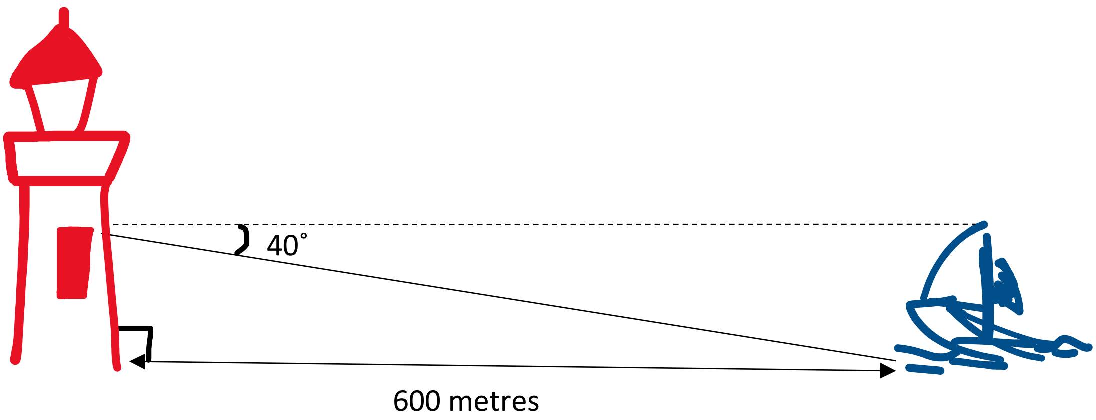
The ship that got nowhere near the lighthouse

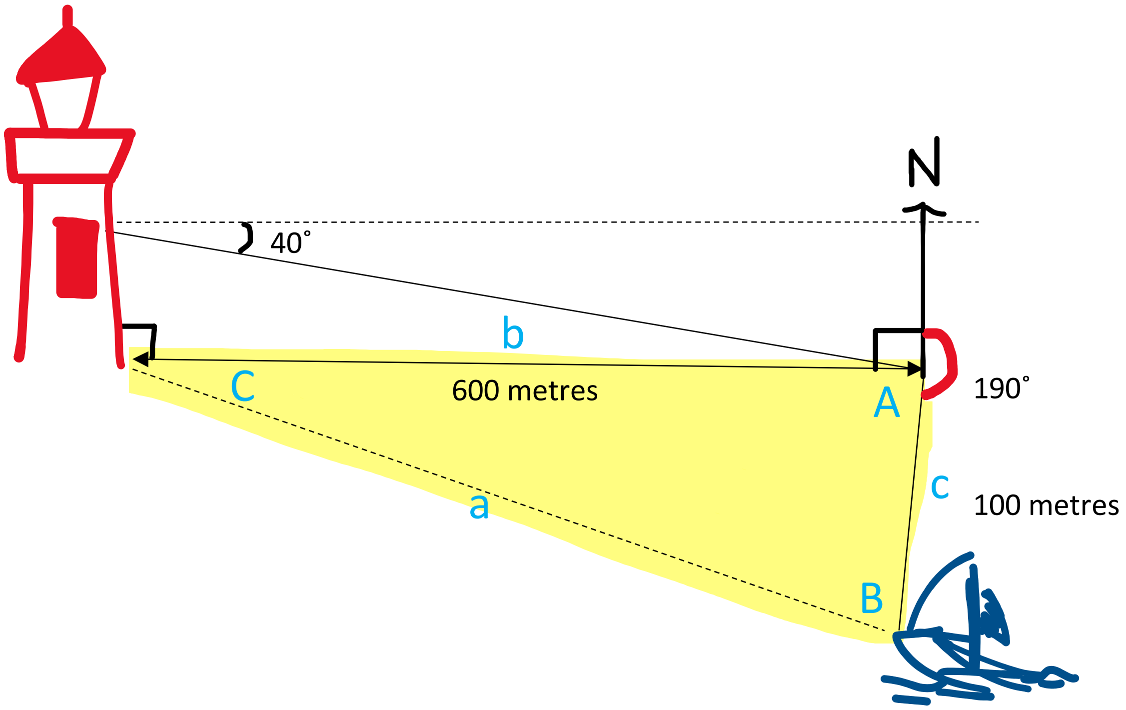
By: Emma Yang

A captain on a ship knows they are close to a lighthouse and the rough direction but can't see it. They are 600 metres away.

A lighthouse keeper at the 10th floor of the light house looks at an angle of depression at 40 degrees and can see the ship but knows that the ship can't see the lighthouse. The angle from the lighthouse to the ground is 90˚.



Suddenly, the area gets foggy, and both the lighthouse keeper and the captain can't see in front of them. The ship now steers off the closest route at a bearing of 190˚, and the distance from where they currently are, and their original position is 100 metres. The new distance from the ship to the bottom of the lighthouse, identified as “a” in the figure below, is:



From the figure, the cosine rule, $a^2 = b^2 + c^2 - (2bc \times \cos(A))$, can be used to find “a”

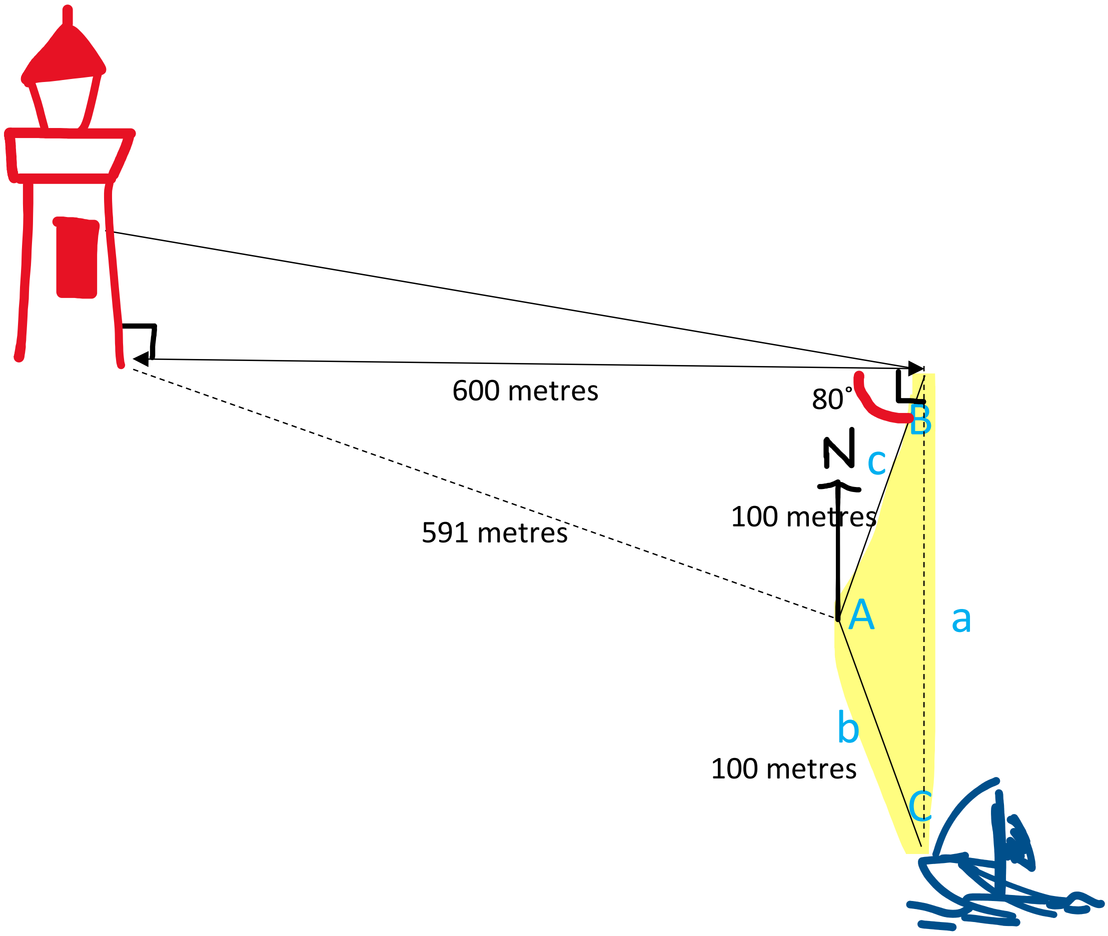
To find the angle A: $360^\circ - 190^\circ - 90^\circ = 80^\circ$

$a^2 = 600^2 + 100^2 - (2 \times 600 \times 100 \times \cos(80))$

a = 591 metres (to the nearest whole number)

The distance between the bottom of the lighthouse to where the ship is now is 591 metres. (which was closer to the lighthouse, but the captain and her sailors didn’t know!)

Not long after, pirates attack the ship. In the end, the captain and her sailors were able to fend off the invaders, but the ship steered 100 metres away because they forgot to put the anchor down when fighting:) Now, they are perpendicular to the original route, and the distance from where they currently are and the original route, identified as “a” in the figure below, is:



The triangle (shaded in yellow) is an isosceles triangle because it has two sides of equal length.

Line a is perpendicular to the original route, so angle B = 90˚ – 80˚ = 10˚

Therefore, angle A = 180˚ – (2 x 10˚) = 160˚

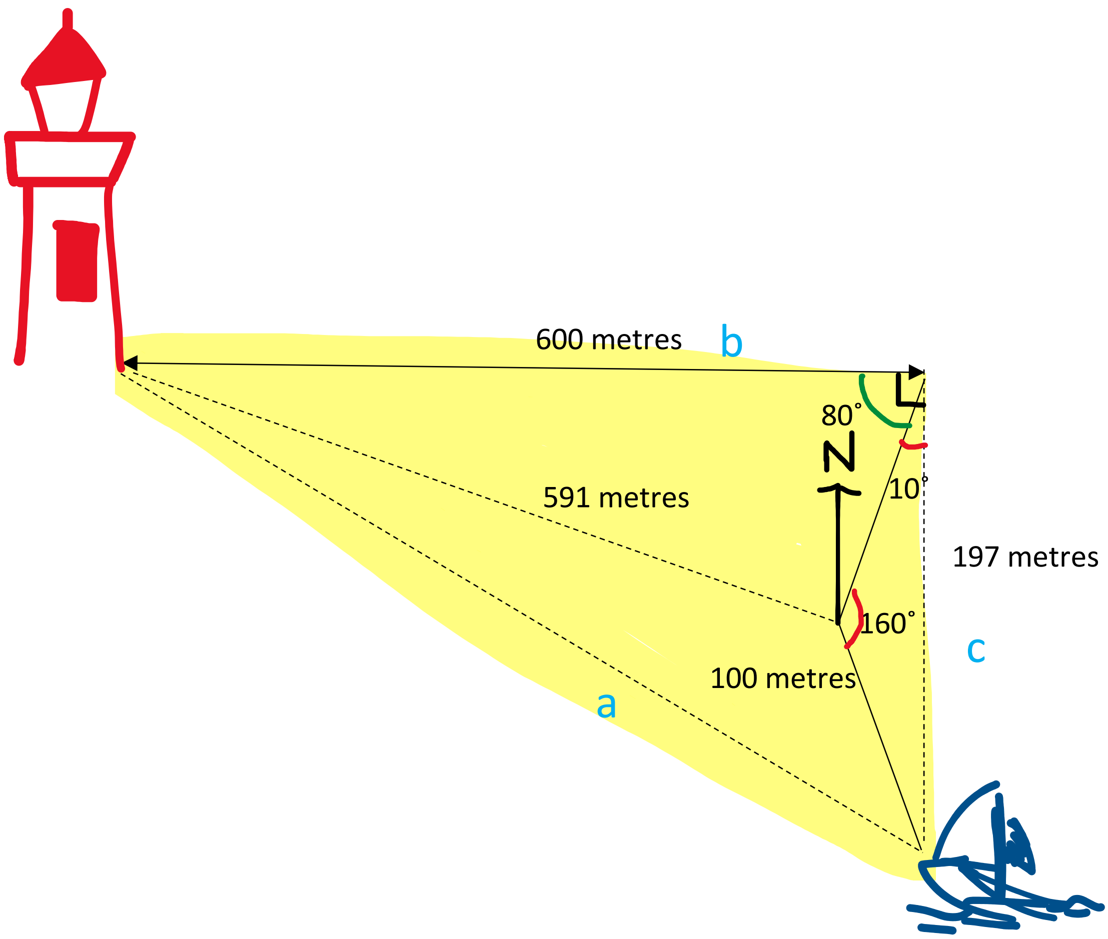
2 angles are known and the angle opposite “a” is known, so “a” can be found using the sine rule: $\frac{\sin{(A)}}{a} = \frac{\sin{(B)}}{b}$

$$\frac{\sin(160)}{a} = \frac{\sin(10)}{100}$$

sin(10) x a = sin(160) x 100

a = $\frac{\sin(160) \times 100}{\sin(10)}$ = 197 metres (to the nearest whole number)

Hope is not lost amongst the captain and her sailors. Once again, they set off in the direction of the original route. Suddenly, one of the sailors thought that it will be faster if they travel straight towards the lighthouse. The distance from where they are now to the lighthouse, identified as “a” in the figure below, is:



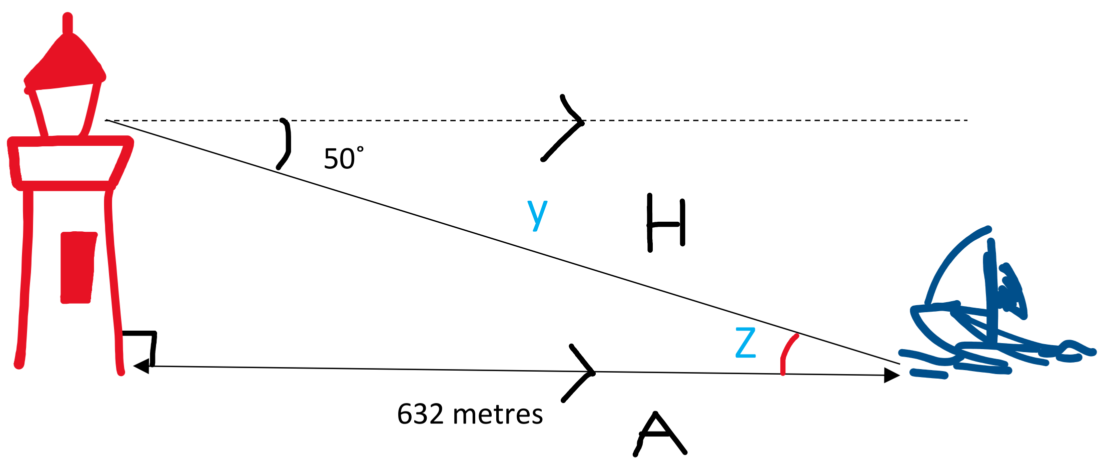
The triangle (shaded in yellow) is a right-angled triangle because line c is perpendicular to line b. Therefore, line a can be found using the Pythagorean Theorem: $a^2 = b^2 + c^2$

$a^2 = 600^2 + 197^2$

$a = \sqrt{600^2 + 197^2}$ = 632 metres (to the nearest whole number)

632 metres < 797 metres (197metres + 600 metres), which means that the sailor is right that going straight towards the direction of the lighthouse is faster than the original route!

The lighthouse keeper is worried about the ship, so he goes to the highest balcony of the lighthouse and is relieved to spot the ship through his binoculars. He estimates (using his 40 years of experience of being a lighthouse keeper) that the ship is 632 meters away, which is still too far away (even further than before) for the ship to see the lighthouse. He is now looking at the ship at an angle of depression of 50˚. The distance from where the lighthouse keeper is standing to the ship, identified as "y" in the figure below, is:



The two horizontal lines are parallel to each other, so the angle of depression and angle Z are alternate interior angles, therefore they are equal.

Angle Z = 50°

To find "y", the function cos (x) = $\frac{A}{H}$ can be used, where 632 metres = A, 50° = x, and y metres = H.

$\cos(50) = \frac{632}{H}$

cos (50) x H = 632

$H = \frac{632}{\cos(50)}$ = 983 metres (to the nearest whole number) = y

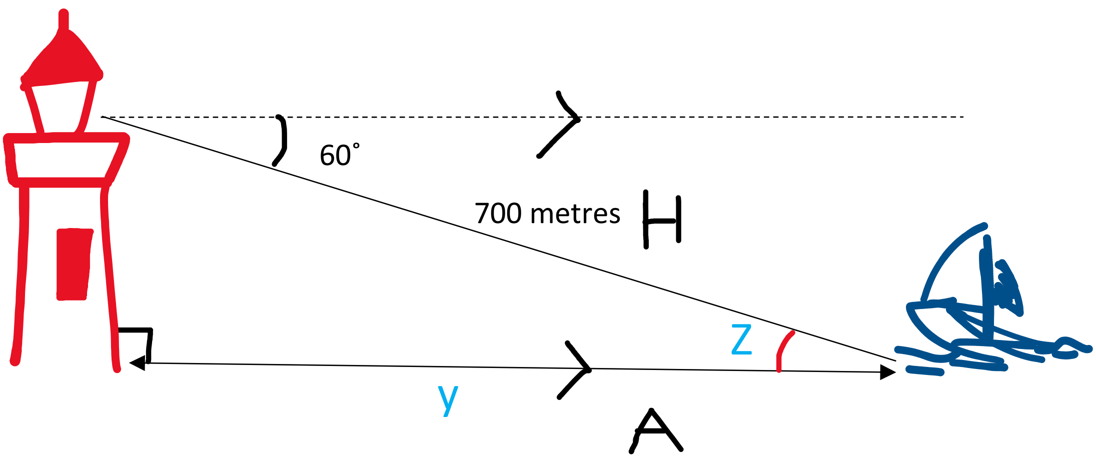
Although they are far away from the lighthouse, the ship is doing successfully in sailing towards their destination and is determined to arrive there after 4 days.

On day 2, the ship realized that their food has run out. One of the sailors thought, as they are on sea, why not catch some fish to eat? (amazing plan) THEREFORE, the ship was stopped (and anchored this time), so the sailors can catch some fish to eat.

Not before long, one of the sailors caught a fish, and a REAL big one. Together, with the other sailors, they pulled, and pulled, and pulled, and pulled, and pulled……and pulled out a gigantic octopus.

Enraged, the octopus monster pounded against the sea surface, making waves that are metres high. The ship was forced off route again.

After that, no one had hope in finding the lighthouse, but not the captain! She encouraged the group of sailors, and again, set off to find the lighthouse. As for the lighthouse keeper, he is now looking down at the ship at an angle of depression of 60˚. He could barely see the ship, but with his brilliant eyesight, he could see a tiny speck far away at a distance of 700 metres, and again, using his brilliant estimation skills, believed that the distance from the bottom of the lighthouse to the ship, identified as "y" in the figure below, is:



The two horizontal lines are parallel to each other, so the angle of depression and angle Z are alternate interior angles, therefore they are equal.

Angle Z = 60˚

To find "y", the function cos (x) = $\frac{A}{H}$ can be used, where y metres = A, 60˚ = x, and 700 metres = H.

cos (60) = $\frac{A}{700}$

A = cos (60) x 700 = 350 metres = y

The ship was 350 metres away from the lighthouse (of course, the captain and the sailors didn't know that)

Luckily, while the octopus was raging, one of his humongous tentacles smashed onto the sharp tip of the ship and was sliced off. This seafood can last for at least 10 days, so food was not a problem anymore.

The ship quickly set off to the direction of the lighthouse again, or so they thought…… because they didn't know that their compass was broken (it had gone through too much adventures)! SO, they rowed, and rowed, and rowed, and rowed…… until the ship reached where it started off, 600 metres away from the lighthouse :)