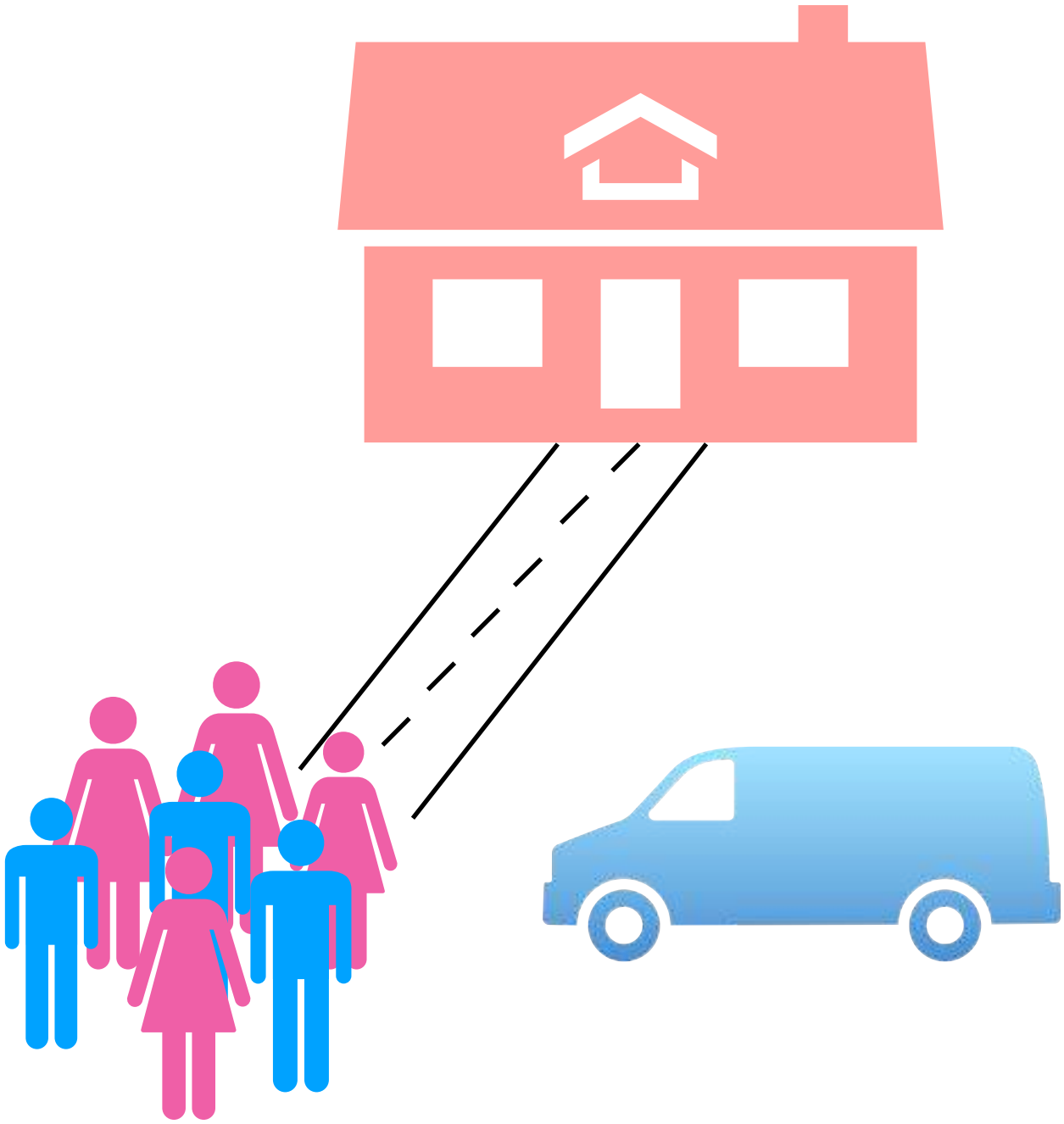
Ramona's Celebration Party



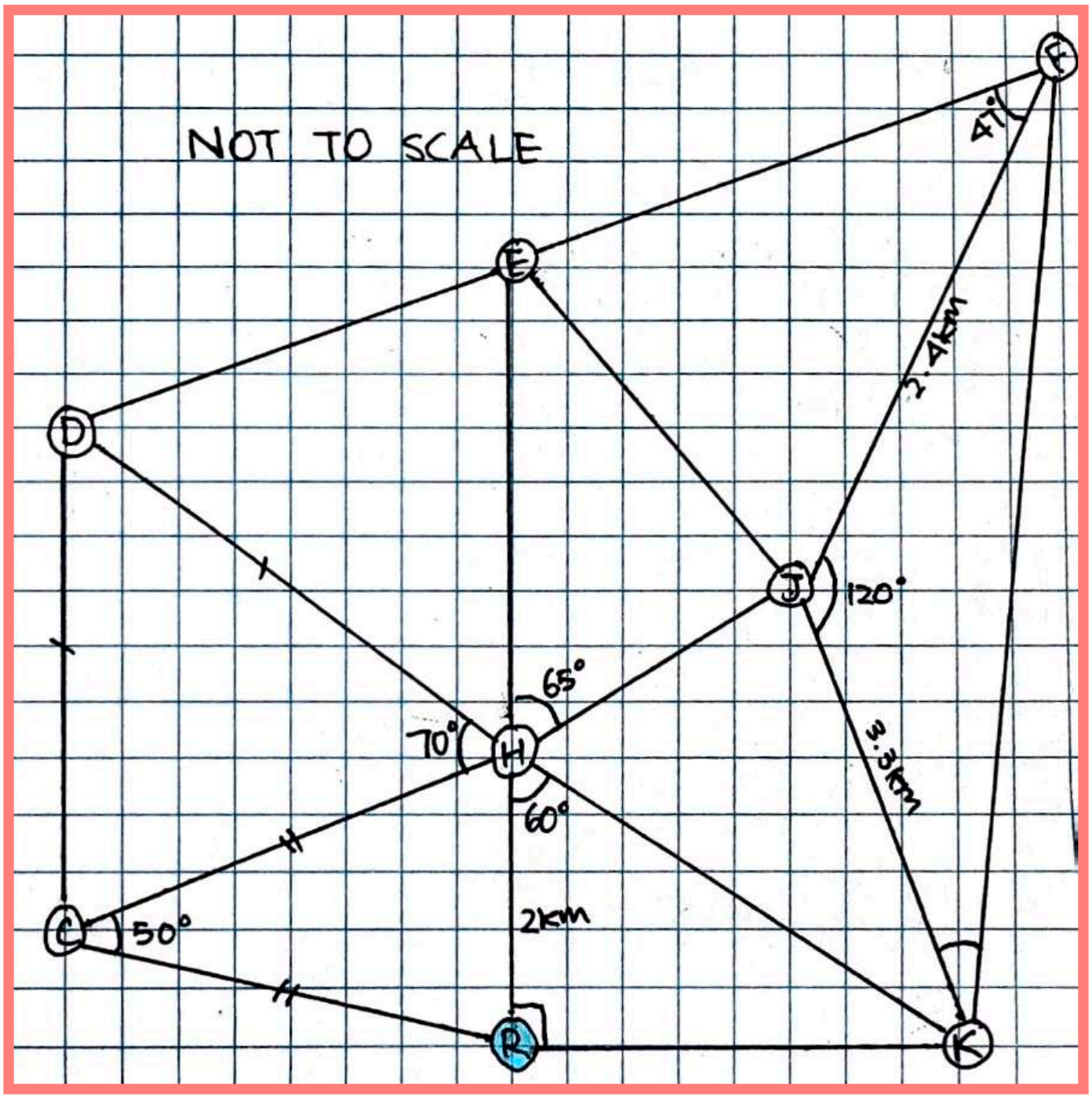
By: Alice Fang

Ramona is throwing a party at her house to celebrate their completion of their final exams. She invited all 7 of her close friends and planned on a carpool.

However, Ramona has just got her new car and she wants to keep her car as new as possible so she tries to keep the car from running unnecessary miles.

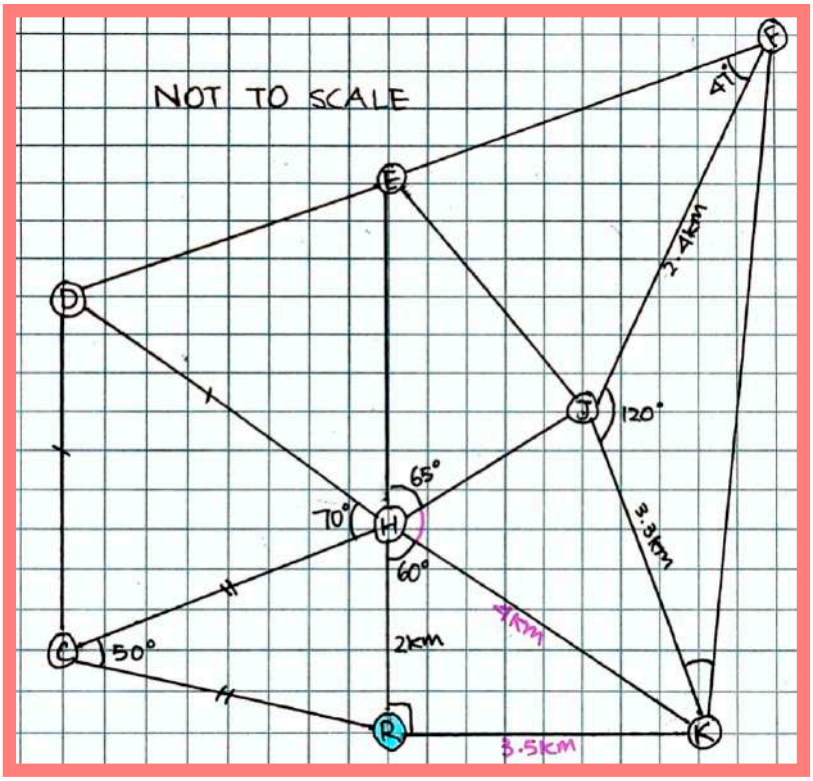
Ramona mapped out all her friends' houses and the possible roads to each of their houses. Each person's first name initial represent their home.

Unfortunately, Ramona only knows some of the measurements. Below is the map Ramona made:



Ramona remembers learning trigonometry in maths class. She going to to calculate the distances between the houses using trigonometry!

Ramona started by calculating the distances between Kathleen, Hailee and her own place.



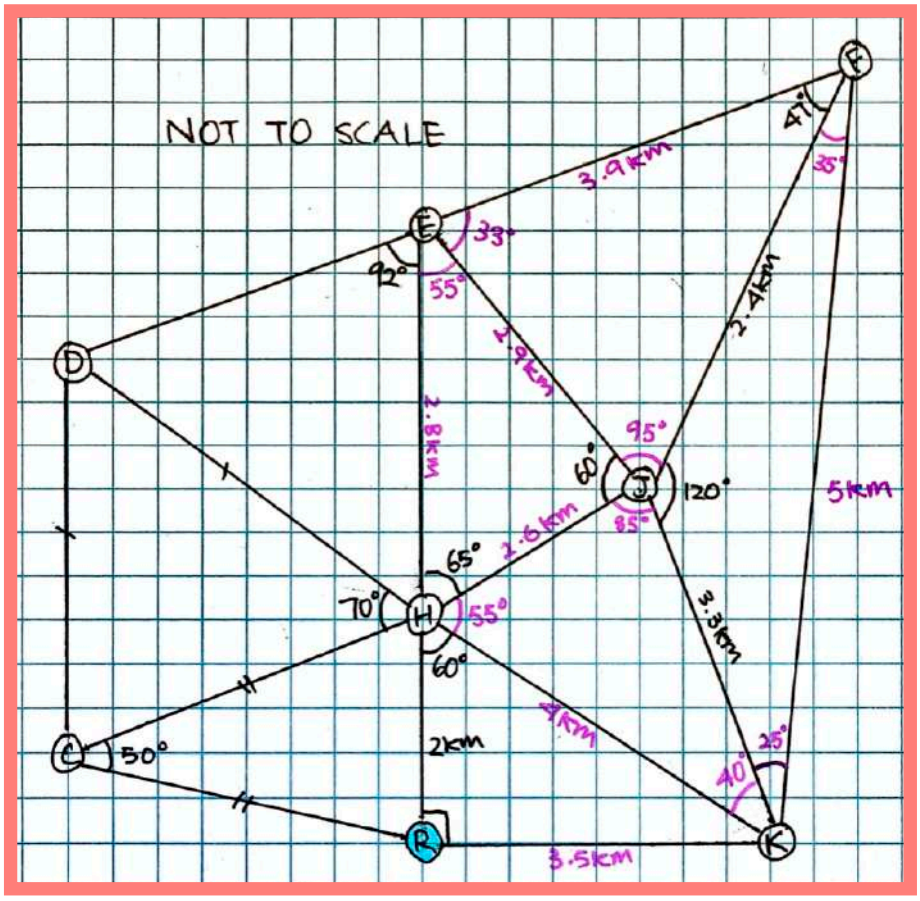
$\overline{RK}$: $\tan 60 \times 2 = 3.5\text{km}$

$\overline{KH}$: $\sqrt{2^2 + 3.5^2} = 4\text{km}$

While she was doing so, she received a text message from Frank. He said that his parents have to leave early for an event and he can't be left alone, so Ramona would have to pick him up first. Ramona agreed and started calculating the distance of all the different paths to go to Frank's house.

Just as Ramona was starting to calculate, Connor called. Connor said he had FOMO (fear of missing out) so he didn't wanted to be in the carpool as early as possible. Ramona decided that she would pick him up after Frank.

Ramona looked at the map she already has:



$\angle JHK = 180 - (60+65) = 55°$

$\frac{\overline{HJK}}{\sin 55} = \frac{3.3}{\sin 55} = \frac{4}{\sin\theta}$

$= \sin^{-1}\left(\frac{4 \times \sin 55}{3.3}\right)$

$= 85°$

$\angle HKJ = 180 - (55+85) = 40°$

$\frac{\overline{HJ}}{\sin 40} = \frac{3.3}{\sin 55}$

$= \frac{3.3 \times \sin 40}{\sin 55}$

$= 2.6\text{km}$

$(\overline{KF})^2 = 2.4^2 + 3.3^2 - 2 \times 2.4 \times 3.3 \times \cos 120$

$\overline{KF} = 5.0\text{km}$

$\frac{\angle JKF}{\sin 120} = \frac{5}{\sin 120} = \frac{2.4}{\sin\theta}$

$= \sin^{-1}\left(\frac{2.4 \times \sin 120}{5}\right)$

$= 25°$

$\angle JFK = 180 - (120+25) = 35°$

$\angle EJF = 360 - (60+85+120) = 95°$

$\angle JEF = 180 - (47+95) = 33°$

$\frac{2.4}{\sin 33} = \frac{EF}{\sin 95} \Rightarrow EF = \frac{2.4 \times \sin 95}{\sin 33} = 3.9\text{km}$

$\angle HEJ = 180 - (65+60) = 55°$

$\frac{2.6}{\sin 55} = \frac{EJ}{\sin 65} = \frac{EH}{\sin 60}$

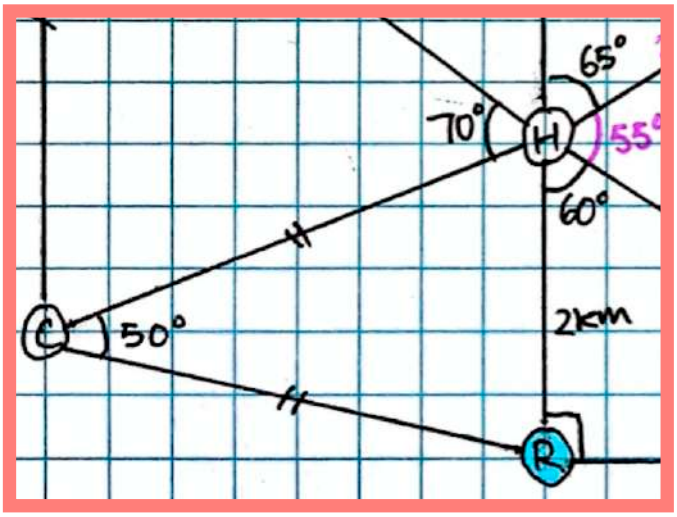
$\overline{EJ} = \frac{2.6 \times \sin 65}{\sin 55}$

$= 2.4\text{km}$

$\overline{EH} = \frac{2.6 \times \sin 60}{\sin 55}$

$= 2.8\text{km}$

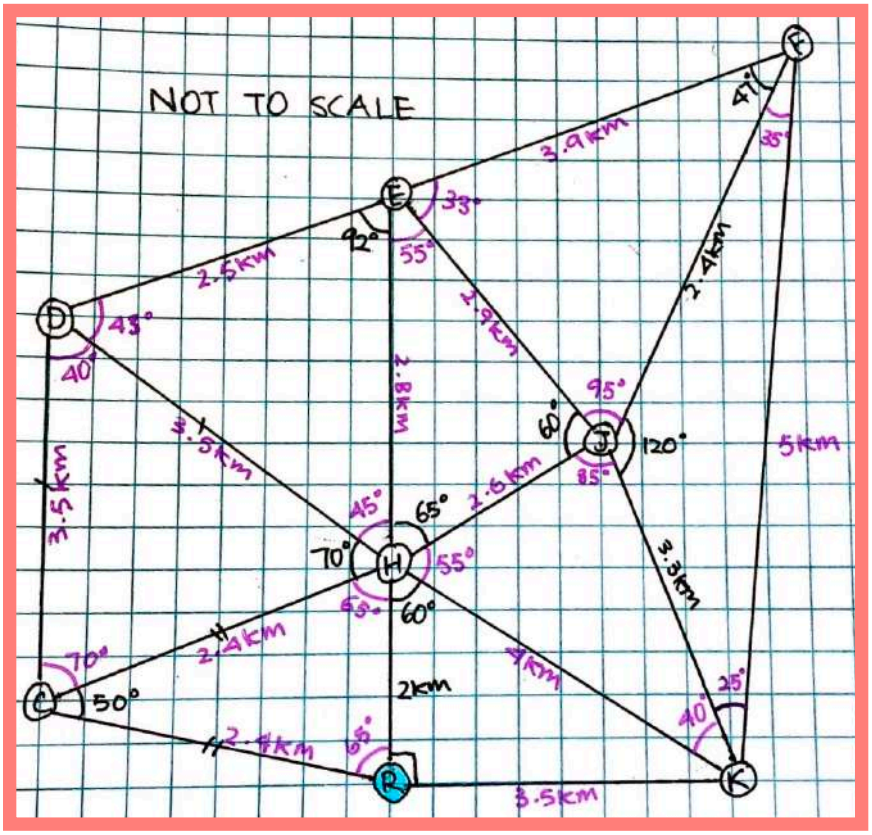
She couldn't figure out how to start calculating the distances between Eileen, Dawson, Connor and her own house! She only knows 3 angles and 2 sides! After deep thought, she realized that there are 2 pairs of parallel lines. She decided that she would start with the triangle pertaining to her house.



For $\triangle RCH$; $\angle CHR = \angle CRH$; $\overline{CH} = \overline{CR}$

$\angle CHR, \angle CRH = (180 - 50) \div 2 = 65°$

$\overline{CH}, \overline{CR} = \frac{2 \times \sin 65}{\sin 50} = 2.4\text{km}$



$\angle CHD = \angle DCH = 70°$

$\angle CDH = 180 - (70 + 70) = 40°$

$\overline{DC} = \overline{DH}$

$\overline{DC}, \overline{DH} = \frac{2.4 \times \sin 70}{\sin 40} = 3.5\text{km}$

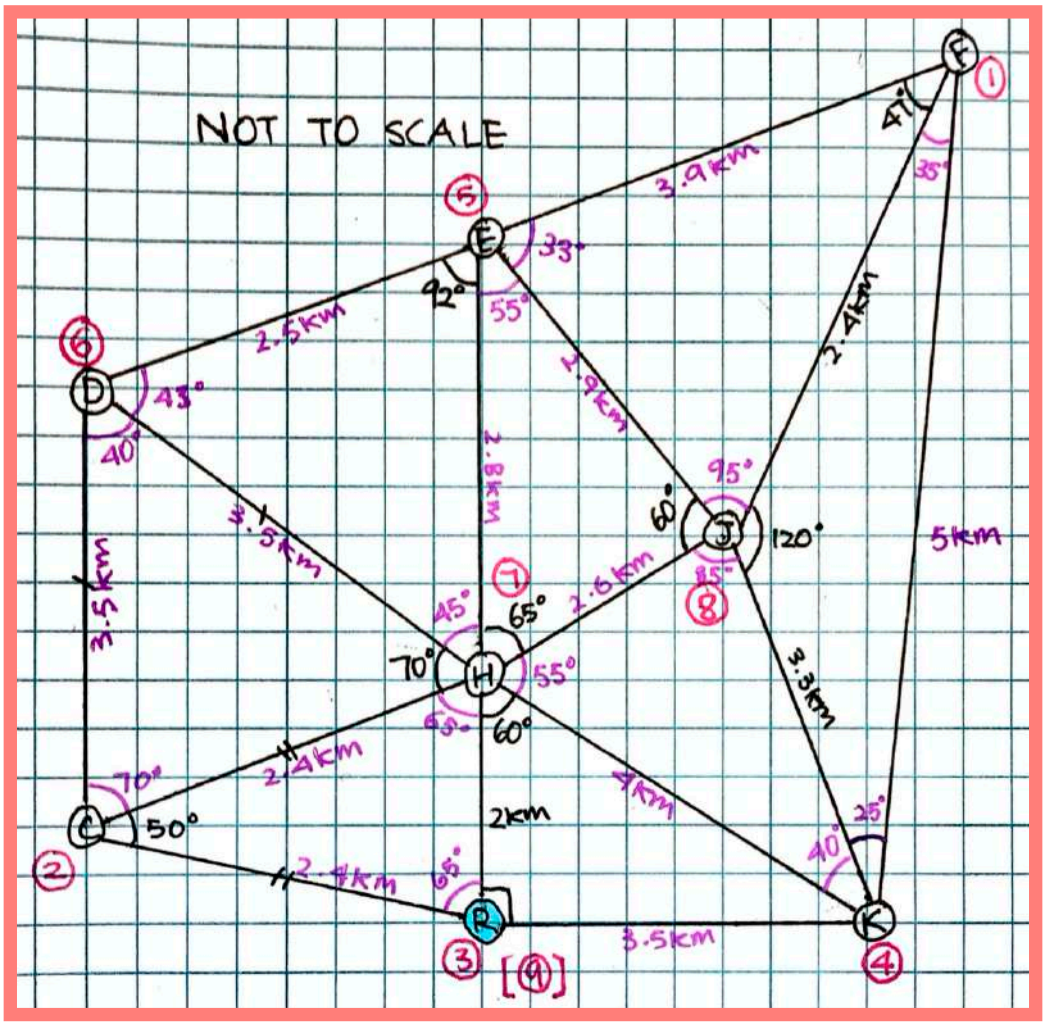
$\angle DHE = 360 - (65 + 55 + 60 + 65 + 70) = 45°$

$\angle HDE = 180 - (45 + 92) = 43°$

$\overline{DE} = \frac{3.5 \times \sin 45}{\sin 92} = 2.5\text{km}$

Ramona's plan is to pick up Frank first then Connor without picking anyone else up on the way because of their requests. She plans to bake a cake, but she has to leave her house before the cake is out of the oven. So she has to return midway or else the cake gets burnt.

Kathleen and Eileen texted Ramona telling her that their younger brothers has a play date with Dawson's younger brother at Dawson's on the same day and asked if their brothers could carpool with them and then drop them off at Dawson's. Ramona was hesitant but generous so she sacrificed her car's extra mileage to pick up Kathleen and her brother after leaving her place to turn off the oven and then head to Eileen's then Dawson's.



After the listings she had to find the shortest distances for all of her paths.

Ramona's → Frank's Pathways

① R → K → F
② R → K → J → F
③ R → H → J → F
④ R → H → E → F

① 3.5 + 5 = 8.5km
② 3.5 + 3.3 + 2.4 = 9.2km
③ 2 + 2.6 + 2.4 = 7km
④ 2 + 2.8 + 3.9 = 8.7km

Frank's → Connor's Pathways

① F → E → D → C
② F → J → H → C
③ F → K → R → C
④ F → E → H → C

① 3.9 + 2.5 + 3.5 = 9.9km
② 2.4 + 2.6 + 2.4 = 7.4km
③ 5 + 3.5 + 2.4 = 10.9km
④ 3.9 + 2.8 + 2.4 = 9.1km

Connor's → Ramona's = 2.4km
Ramona's → Kathleen's = 3.5km

Kathleen's → Eileen's Pathways

① K → J → E
② K → H → E
③ K → R → H → E

① 3.3 + 2.9 = 6.2km
② 4 + 2.8 = 6.8km
③ 3.5 + 2 + 2.8 = 8.3km

Eileen's → Dawson's = 2.5km
Dawson's → Hailee's = 3.5km
Hailee's → Justine's = 2.6km

Justine's → Ramona's Pathways

① J → K → R
② J → H → R

① 3.3 + 3.5 = 6.8km
② 2.6 + 2 = 4.6km

So the total distance Ramona will travel is:

7 + 7.4 + 2.4 + 3.5 + 6.2 + 2.5 + 3.5 + 2.6 + 4.6 = 39.7km

After Ramona's careful planning, she picked up all her friends driving minimum distance while fulfilling everyone's needs. Ramona and her friends all had a great time and enjoyed the delicious cake Ramona baked! Kathleen, Eileen and Dawson's brothers also had a great time at their play date!

THE END

Ramona is hosting a party with her friends at her place to celebrate the completion of their final exams. She has to carpool all her friends to her place, but she encounters problems and she wants to keep her new car from running unnecessary distance...

What is she going to do?