THE 2023 YOUNG MATHEMATICAL STORY AUTHOR (YMSA) COMPETITION

THE CINDY NEUSCHWANDER AWARD

(THE 12-15 YEARS OLD CATEGORY)

LONGLISTED

'A Sweet Summer' by Sophie Wang (13 years old)

at Fulton Science Academy (Georgia, USA)

You can read the author's inspiration for the story and the judges' comments on:

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#YMSAMaths

a sweet summer

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Ava and Isabella were classmates and besties. They just graduated from Creekside Elementary School. End of the elementary era and summer break was finally here!

Both girls loved animals. One of their favorite places to volunteer was called Furball, which was an animal shelter for all sorts of abandoned pets. In addition to volunteering at Furball during the summer break, Ava and Isabella wanted to do more for Furball. And they decided to raise money for their fur friends there.

After brainstorming for a couple of days, the girls came up with the idea of a lemonade stand. Selling lemonade in their subdivision's swimming pool and tennis court area sounded like a fun and feasible fund-raising activity. And if they could raise $300, that would be awesome.

When their parents heard about the lemonade stand, they immediately agreed that it was a great idea.

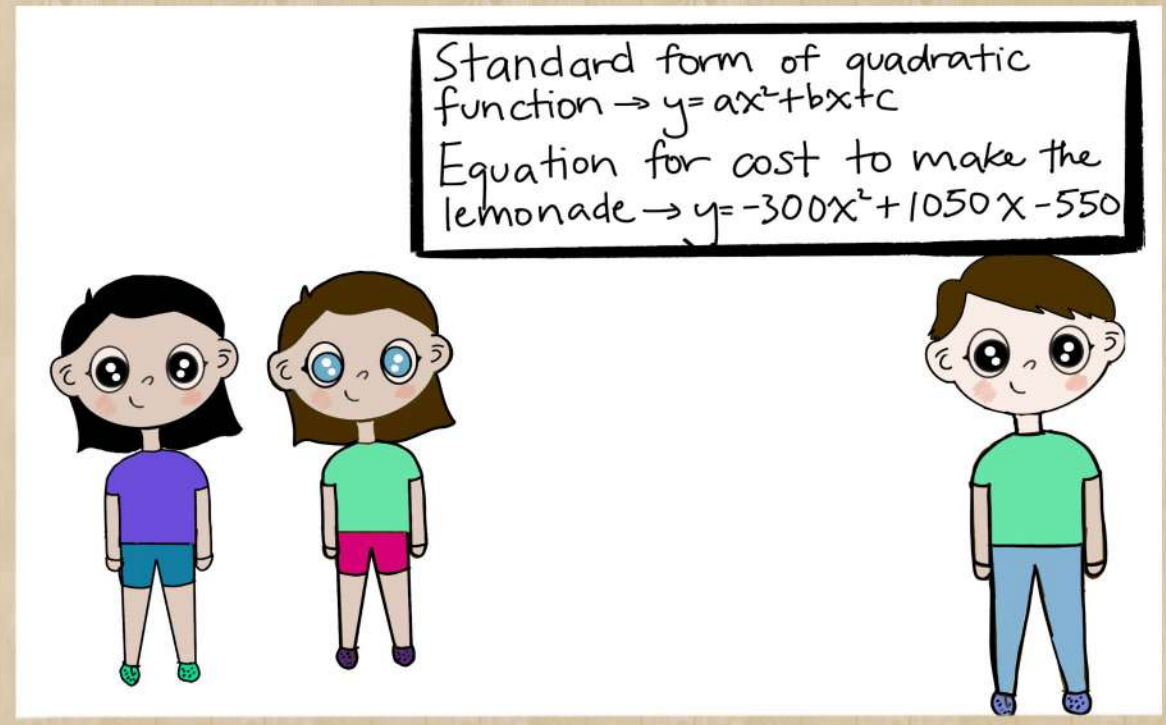
"What a great way for the girls to earn their first dollars!" Ava's dad remarked."I'm sure the kids from the swimming pool and the tennis courts would love to have a cup of cold lemonade!" Isabelle's mom smiled and commented.

"And the animals at Furball will thank you girls for sure!" Isabella's dad added.

Running a lemonade stand, however, wasn't as easy as they had expected. There were a lot of things to consider and plan, since the girls had never run a fund-raising activity before.

What supplies do they need? Where can they find a good lemonade recipe? And most importantly, how much should they charge for a cup of lemonade? They knew it couldn't be too expensive or too cheap. It had to be the right price for them to make money and cover all the costs at the same time. But how could they set the price? After struggling with it for a day, they went to ask Ava's high school brother, Andrew to help them. Andrew happily agreed to help them. He really liked math and it was satisfying to him if he could explain a math problem or topic to someone else.

After listening to the lemonade stand idea and their goal to raise at least $300 for Furball, Andrew said, "You probably need to use some math knowledge in your fund- raising planning!" "We both love math!" exclaimed Ava. "But how is math going to help us?" Isabella was a little doubtful. "Have you ever heard of quadratic functions? Andrew started to throw out math terms, with a big grin. "What's a quadratic function?" Ava and Isabelle both asked.



"A quadratic function is a function which has one variable and the highest degree is 2. The standard form of a quadratic function is y= ax^2 + bx + c where a, b, and c are numbers and x is the variable. When graphed, a quadratic function forms a parabola which is a symmetrical curved line. When a is positive, the quadratic function opens up and looks like a smile, when a is negative it opens down like a frown." Andrew talked and wrote the function down on the white board in his room. He also graphed a sample quadratic function as he was talking, as if he was a teacher to Ava and Isabella. He made sure that they followed what he was talking about before moving on.

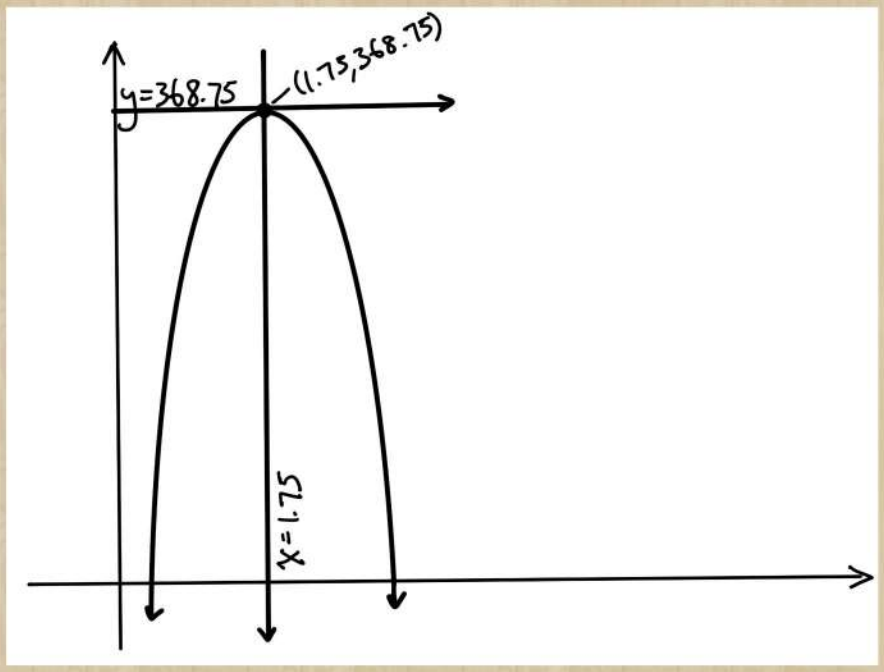
The girls were smart, and fast learners. Even though they just graduated from the 5th grade, they understood basic algebra enough to get what Andrew was talking about.

"For your lemonade business, if all variables are set correctly, the highest point of the parabola or the graph is the price at which you will make the highest profit. However, in order to derive the function for the business, we need to set some variables or numbers. For example, the setup cost for the lemonade stand, the cost to make each cup of lemonade and the number of cups of lemonade you plan to make."

"Ok," Isabella replied, "We already spent about $100 to set up the lemonade stand, all supplies included. The supplies allow us to make up to 900 cups of lemonade. Each cup is 12 fluid ounces and it would cost us roughly $0.50per cup, based on our calculation."
"Alright, great." Andrew continued, "Based on the marketprice of a cup of lemonade, no one will buy any lemonade from you if the price is $3 per cup. On the opposite side, if the price is $0, you will be giving away all of the lemonade. A simplified way to estimate the number of cups sold is equal to 900-300x, x being the price per cup of lemonade. To go from the number of cups to the profit made, we need to use the total revenue and expense. You total revenue is (900-300x)*x and the total expense is (900-300x)*0.5 +100. Profit is equal to revenue - expense. So the profit is y=-100+(900-300x)(x-0.5).
"Ok," Isabella said, "This actually does make sense $100 dollars is the money we spent on setting up the lemonade stand, the (x-0.50) is the price per cup minus the cost per cup or the profit each cup, and the (900-300x) are the cups of lemonade sold?"
"Yeah, Isabella, that's exactly what all of those mean! Wow, you guys are fast learners!" Andrew exclaimed.

- 100 dollars set up
- 900 12 fluid ounce cups
- Cost 50¢ to make each cup
- $900 \times 0.5 + 100 = 550$
- Costs a total of 550 dollars to set up the lemonade stand

"Let's continue, now all we need to do is to solve for x, the price of each cup of lemonade. Can you girls try to simplify the function by combining similar terms first?"



Andrew posed a question to Ava and Isabelle.

"Sure," Ava said, "First, we distribute so we get $y=-100+900x-450-300x^2+150x$. Then we can combine the like terms: $y=-300x^2+1050x-550$ This is the quadratic function for our lemonade business, right Andrew?"

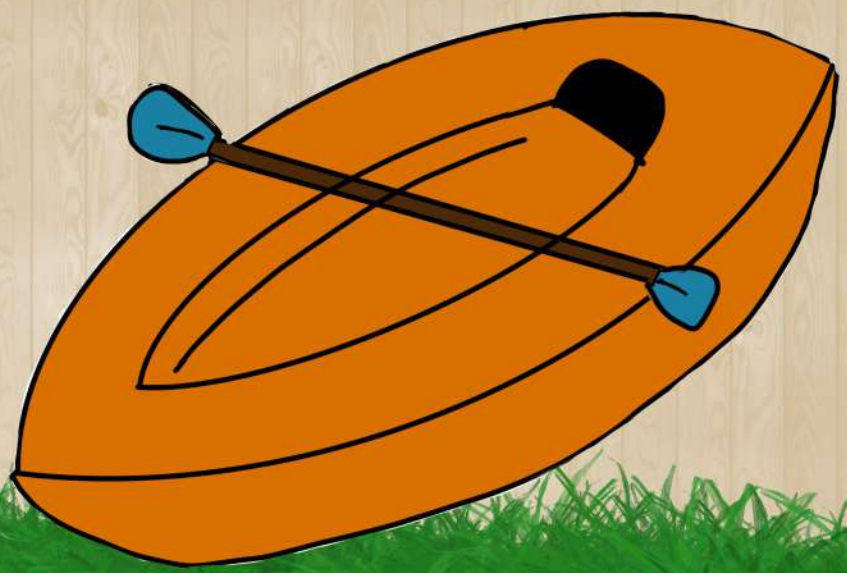
"Yeah, that's correct, good job Ava," Andrew responded. "This quadratic function has a parabola facing down because it has a negative a. The vertex is the highest point and the point where you can achieve the most profit. To get the vertex, you'll need to find the axis of symmetry using the formula $x=-b/2a$, which in this case is $-1050/-600 = 1.75$. Then you can plug the $x=1.75$ in for the value of y. And you'll find that $y=386.75$. The vertex of this quadratic function is (1.75,386.75). This means that you will make $368.75 of profit if you sell each cup of lemonade at $1.75, and if you sell 900-300 $x=375$ or 375 cups of lemonade."

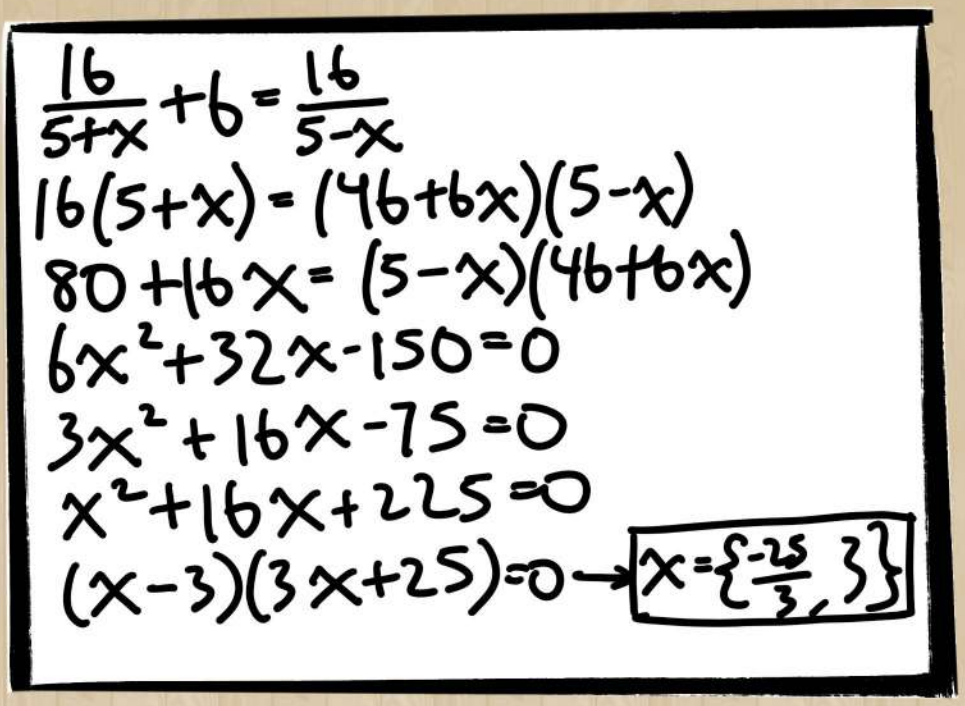
"Wow, that is a lot of money!" Isabella exclaimed.

"Yes, did that all make sense?" Andrew asks.

"Yes, that's so cool how quadratic functions are used to solve a real-life problem!" Ava exclaimed.

"375 cups of lemonade are a lot to make and to sell. We need to get going right away. If we work a full week or 7 days, that's a little over 50 cups per day, which does not sound too bad at all!" Isabelle added.
"If you guys succeed in selling all the 375 cups as shown by the quadratic equation, you'll have $86.75 left after your donation to Furball. Do you have any plans to reward yourself for your hard work?" Andrew posed another question.
"Well, Isabelle and I always wanted to go kayaking. Can we all go together since you helped us with our lemonade business?" Ava asked Andrew.
"I know a very good place for us to go kayaking but I want to give you guys another real-life problem to see if you really understand quadratic equations. If you guys get it, I'd go with you on the kayaking trip. Sounds fair?" Andrew asked the girls.
"Alright," Ava said, "We are ready!"
"Ok," Andrew replied, "The problem is that you two are going kayaking. The river is 16 miles long. You guys can go 5 mph when the water is still. When you row upstream, it takes them 6 hours longer than when they row downstream. You row a round trip. What is the speed of the current?"



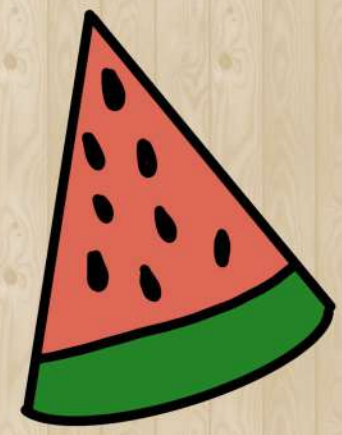


"Oh," Isabella says, "Let me see. I have an idea. We set up an equation using the relationship between the time it takes to go upstream and the time it takes to go downstream. When the kayak goes downstream at the same direction at the current, the speed is 5+x. Similarly, the speed to go upstream is 5-x. The difference between the hours taken is 6 hours. She wrote the following work on the board.

16/(5+x)+6=16/(5-x)

"But how do you solve this, Andrew?" Isabelle murmured.

"I'm so glad you are able to set up the formula for the kayaking problem. It tells me that you already understood the basic application of the quadratic functions. Regarding solving the quadratic function, you will learn this soon in your middle school math classes, but I can show you what it looks like now and also help you to find the solution." Andrew said excitedly.



Summer break was waiting for Ava and Isabelle, who were recent elementary school graduates. They were brainstorming something big. Yes, a fund-raising event for Furball, an animal shelter. The girls got stuck on how to set the price of a cup of lemonade for their brand-new lemonade stand business. Andrew was pulled in to help. Was he a great math teacher! How were boring math equations going to help the girls? And did the girls plan for anything fun after the fund-raising event? And how was math going to help them again, even in their kayaking trip?

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